Corrections to the book "Quantum calculus" by Victor Kac and Pokman Cheung

1. page 11. In the formula above (3.10) instead of $(a - q^{n+1}x)$ should be $(a - q^n x)$

2. page 12. In the example should be $[n]D_q^{j-1}x^{n-1} = [n][n-1]D_q^{j-2}x^{n-1}$

3. page 24, last paragraph. The proof is incorrect, replace it by the following argument: A linear transformation T of rank j is specified by the subspace K of A, mapped by T to 0, and by a collection of j linearly independent vectors b_1,..., b_j in B such that $T(a_i) = b_j$ for a fixed collection of vectors a_1,...,a_j, which together with a basis of K form a basis of A. This proves the desired formula for the jth summand.

4. page 27, last line. Add after $a = 0$: Suppose, in addition, that the degree of the lowest degree monomial in $P_j(x)$ tends to ∞ as j tends to ∞.

5. page 28, lines 5-7.The beginning of the proof of Theorem 8.1 should read: It is easy to see that for any formal power series $f(x)$, we have

6. page 42, in the second line of formulas the symbol $\sum_{m|n}$ should be deleted (twice).

7. page 52, line 11. Instead of $n \to -\infty$ should be $n \to \infty$

8. page 66, lines 7,8. Should be: This formula means that $F(u(x))$ is a $q^{1/\beta}$-antiderivative of $f(u(x))D_{q^{1/\beta}}u(x)$.

9. page 74. Corollary 20.1 and its proof should be replaced by the following:

   Corollary 20.1. If $f(x)$ is continuous at $x = 0$ and $x^\alpha D_q f(x)$ is bounded on the interval $(0, A)$ for some $0 \leq \alpha < 1$ and $A > 0$, we have for $a, b \in [0, A]$:

$$\int_a^b D_q f(x) d_q x = f(b) - f(a) \tag{20.2}$$

   Proof. By the discussion after Theorem 19.1, $f(x)+$ const are all antiderivatives of $D_q f(x)$, which are continuous at 0, and, by Theorem 19.1, one of them is given by Jackson integral (19.2). Now we can apply Theorem 20.1. □

10. page 76. There are divergence problems with the definition (21.6) of the q-gamma function because the function E_q^{-x}, surprisingly, blows up along some sequences as x tends to infinity. (It is because the radius of convergence of the series (9.7) for e_q^x is $1/(1-q)$, not infinity.) However, if we replace the upper limit of the integral (21.6) by $1/(1-q)$, this difficulty is removed, but all arguments on page 77 still hold with little modifications, given below.

11. page 76, line 4 from the bottom. Add: Then we have: $[\infty] = 1/(1-q)$.

    The upper limit of the integral in the definition (21.6) of the function $\Gamma_q(t)$ should be $[\infty]$ instead of ∞

12. page 77. The first sentence should read: First we note that by (9.10), $E_q^0 = 1$ and $E_q^{-[\infty]} = 0$.

    The upper limit of the integral in lines 3 and 7 should be $[\infty]$ instead of ∞.

    The sentence after the definition of the q-beta function should read:

    By the definition of the q-integral (19.7), we have

    In the line that follows the letter a should be removed, the next line should be removed, and in line 9 from the bottom the upper limit of the integral should be 1 instead of ∞. The line after that should be removed.

    The upper limit of the integral in line 5 from the bottom should be 1 instead of ∞.

    In line 4 from the bottom should be (19.14) instead of (19.15).

    In line 3 from the bottom the upper limit should be $[\infty]$ instead of ∞.

13. page 79. Instead of the sentence Then both sides are formal power series in q. should be

    Then both sides are formal power series in two variables q and $v = q^t$.

14. page 80. At the end of the first paragraph add:

    We shall assume that $h > 0$.

15. page 81. In Example replace $(x+b)^N$ by $(x+b)_h^N$ (twice).

16. page 83. In line 6, instead of $D_h(F(x)g(x))$ should be $D_h(f(x)g(x))$

    In line 8 from the bottom, instead of $a < b$ should be $0 \leq a < b$

    In the subsequent definition of $f(x)$ add that $f(0) = 0$

17. page 84. In line 13 after $h > 0$ add and $x > a$

    In formula (22.19) replace $|x-a|^{n+1}$ by $(x-a)_h^{n+1}$

18. page 103. It is not true in general that the polynomials $P_n(x)$ have the form (26.20). Therefore one has to use the following generalization of Theorem 2.1.

    Theorem 26.2 *Let a, q be some numbers, D be a linear operator on the space of polynomials, and $\{P_0(x), P_1(x), \ldots\}$ be a sequence of polynomials, satisfying three conditions:*

    (a) $P_0(a) = 1$, $P_n(a) = 0$ *if n is odd, and* $P_n(qa) = P_n(q^{-1}a) = 0$ *if n is positive even;*

(b) *deg* $P_n(x) = n$;

(c) $DP_n(x) = P_{n-1}(x)$ *for any* $n \geq 1$ *and* $D(1) = 0$.

Then for any polynomial $f(x)$ one has:

$$f(x) = \sum_{n \geq 0 \text{even}} (D^n f)(a)(q^{-n} P_n(qx)) + \sum_{n > 0 \text{odd}} (D^n f)(q^{-1}a) P_n(x) \,.$$

The proof of this theorem is the same as that of Theorem 2.1. However, unlike Theorem 2.1, Theorem 26.2 can be applied to the operator $D = \tilde{D}_q$ and the polynomials $P_n(x) = (x-a)^n_{\tilde{q}}/[n]^{\sim}!$.

Moreover, using the same argument as that in the proof of Theorem 20.2, one can derive a similar q-analogue of Taylor's formula with the Cauchy remainder in the symmetric q-calculus.