Harvard-M.I.T. Algebraic Geometry Seminar

TORIFIED SIMPLICIAL COMPLEXES FROM SAMUEL-REES DEGENERATIONS

Allen Knutson
U. C. Berkeley

Let X be a projective variety, and H a hyperplane section of it, possibly nonreduced. We warm up with three proofs of the equation $\deg(X) = \deg(H)$: (1) "it's obvious", (2) "using the associated graded to the (b)-adic filtration, degenerate X to a cone on H", (3) "using the associated graded to the Samuel homogenization of the (b)-adic filtration, degenerate X to a weighted cone on the reduction of H." (I won't assume anyone's heard of these Samuel filtrations before.)

This last is the most geometrically satisfying, and if we iterate it, we get a reduced union of weighted cones on weighted cones on weighted cones ... on points. Each component of the limit normalizes to a toric variety, and they are glued along toric subvarieties. In particular the coordinate ring comes with a great basis, where any product is either 0 or another basis element.

I'll focus on the case where X carries a circle action with isolated fixed points, which gives a canonical way to choose the bs; for X a flag manifold this reproduces the Littelmann path model (but without all that representation theory). I'll also talk about the case $X = \overline{M_{0,n}}$.

Tuesday, February 15th, 2005
3:00 p.m.
MIT Room 4–163

http://www-math.mit.edu/ags/