Harvard-M.I.T. Algebraic Geometry Seminar

ELEVATED RANK IN POSITIVE CHARACTERISTIC: GEOMETRIC ASPECTS

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For a global field K and an elliptic curve E over $K(T)$, Silverman's specialization theorem implies the rank of $E_t(K)$ is at least as large as the rank of $E(K(T))$ for all but finitely many $t \in \mathbf{P}^1(K)$. If this inequality is strict for all but finitely many t, the elliptic curve E is said to have elevated rank. All known examples of elevated rank for $K = \mathbf{Q}$ rest on the parity conjecture for elliptic curves over $\mathbf{Q}$, and the examples are all isotrivial.

Some additional standard conjectures over $\mathbf{Q}$ imply that there does not exist a non-isotrivial elliptic curve over $\mathbf{Q}(T)$ with elevated rank. In positive characteristic, an analogue of one of these additional conjectures is false. Inspired by this, for the rational function field $K = k(u)$ over any finite field k with characteristic not 2, we construct an explicit 2-parameter family $E_{c,d}$ of non-isotrivial elliptic curves over $K(T)$ (depending on arbitrary non-zero c and d in k) such that, under the parity conjecture, each $E_{c,d}$ has elevated rank. The proof rests on a combination of geometric, arithmetic, and cohomological arguments.

This is joint work with K. Conrad and H. Helfgott. We will emphasize the geometric aspects of the work, and some other aspects will be discussed in K. Conrad's talk at BU on October 18 (but it will not be assumed that the audience goes to both talks).

Tuesday, October 19th, 2004
3:00 p.m.
Harvard Rm 507

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