1 Introduction

My research lies at the intersection of noncommutative algebra, representation theory, and combinatorics and is focused on the study of quantum Schubert cells $U^+[w]$ introduced by De Concini, Kac, Procesi [8] and by Lusztig [24]. There is one such algebra for each simple Lie algebra $\mathfrak{g}$ and Weyl group element $w$. Lusztig’s braid group action on $U^+_w(\mathfrak{g})$ and a chosen reduced decomposition of $w$ give rise to a distinguished generating set for $U^+[w]$. For example, if $\mathfrak{g}$ is $\mathfrak{sl}_2$, the Lie algebra of traceless $2n \times 2n$ matrices, then the decomposition

$$(s_n s_{n-1} \ldots s_1)(s_{n+1} s_n \ldots s_2) \ldots (s_{2n-1} s_{2n-2} \ldots s_n)$$

of the associated permutation on $2n$ elements gives rise to the algebra of $n \times n$ quantum matrices.

These algebras support a natural algebraic torus $H$ action, and their noncommutative prime ideals $\text{Spec}(U^+[w])$ and $H$-prime ideals $H\text{-Spec}(U^+[w])$ have strong ties to Poisson geometry, Weyl group combinatorics, quasideterminants, quantum cluster algebras, total positivity, Khovanov–Lauda–Rouquier algebras, and canonical bases and perverse sheaves [6, 7, 13, 16, 24, 26, 30].

In 2003, Cauchon proposed an abstract procedure to eliminate complicated commutation relations in $U^+[w]$ yielding a quantum affine space $U^+[w]$. The image of the induced embedding $\text{Spec}(U^+[w]) \rightarrow \text{Spec}(U^+[w])$ can then be stratified and parametrized by $H$-primes. Describing $U^+[w]$ is very difficult, but the following theorem yields a concrete formulation:

**Theorem 3.1** (G., Yakimov). Up to a multiple of $(q^{-1}_{a_i} - q_{a_i})^{-1}$, the generators of $U^+[w]$ for Cauchon’s procedure applied to $U^+[w]$ are quantum minors or quotients of quantum minors [17].

Using representation theoretic methods involving Demazure modules and quantized coordinate rings, Yakimov [32] constructed a poset isomorphism between $H\text{-Spec}(U^+[w])$ and the Bruhat order interval $W^\leq w$. Alternatively, Cauchon and Mériaux [7] utilized combinatorial methods to construct a mere set bijection $H\text{-Spec}(U^+[w]) \rightarrow W^\leq w$. Yakimov and I answered two natural questions posed by Cauchon and Mériaux [7 5.3.2 and 5.3.3], thereby unifying the seemingly disparate methods:

**Theorem 3.2** (G., Yakimov). The Yakimov and the Cauchon–Mériaux methods coincide [17].

To illustrate the power of combining representation theoretic and ring theoretic approaches, we offer a new and simple proof of the Cauchon–Mériaux classification of $H\text{-Spec}(U^+[w])$ [17].

I expect to extend similar techniques to other classes of algebras including Manin’s super quantum matrices. Backed by much evidence, I aim to discover a non-recursive quasideterminant formula for Cauchon’s algorithm (see Section 4.1). This would suggest a method for investigating $\text{Spec}$ of more general algebras. I am also working on a categorification of quantum Schubert cells that will align with the KLR one when $w$ is chosen to be the longest word (see Section 4.2). Discussions with Lauda, Muller, and Webster suggest methods for completing this.

My research is not limited to quantum groups. Mahlburg, Muller, Thurston, and I are investigating the superunital domains of cluster algebras $\mathcal{A}$—regions in $\mathbb{R}_+^n$ with striking similarities to the generalized associahedra of Fomin and Zelevinsky when $\mathcal{A}$ is finite type. The quotient $\mathcal{S}_\mathcal{A}$ of the superunital domain by the action of the Teichmüller modular group of $\mathcal{A}$ supports a natural logarithmic volume. Under certain restrictions on $\mathcal{A}$, this volume over $\mathcal{S}_\mathcal{A}$ seems to always be a rational multiple of the Riemann zeta function (see Section 4.3). In a project with Bremer, Hoffman, and Muller, I am working on $GKZ$ holonomic systems and Gauss–Manin connections. We explicitly calculate Picard–Fuchs equations for one-parameter families of Calabi–Yau hypersurfaces arising from simply-laced root systems. As an unintended consequence, we obtain previously unknown recursive formulas for numbers of loops of length $n$ in root lattices (see Section 4.4).
2 Some Background on $\text{Spec}(U^{\pm}[w])$

2.1 $\mathcal{H}$-stratification of noncommutative prime spectra

If $\mathcal{H}$ is a group acting rationally by automorphisms on $R$, we say the ideal $I$ is $\mathcal{H}$-invariant (or an $\mathcal{H}$-ideal) if $h(I) = I$ for all $h \in \mathcal{H}$, and we call a proper $\mathcal{H}$-ideal $P$ an $\mathcal{H}$-prime ideal of $R$ if the product of two $\mathcal{H}$-ideals being in $P$ implies at least one is in $P$. We denote by $\mathcal{H}$-$\text{Spec}(R)$ the collection of $\mathcal{H}$-prime ideals of $R$. The action of $\mathcal{H}$ on $R$ induces an action on $\text{Spec}(R)$, thereby partitioning $\text{Spec}(R)$ into (possibly infinitely many) $\mathcal{H}$-orbits. Following Goodearl and Letzter [15], this partition can then be refined to the $\mathcal{H}$-stratification of $\text{Spec}(R)$

$$\text{Spec}(R) = \bigsqcup_{J \in \mathcal{H} \text{-Spec}(R)} \text{Spec}(J)(R),$$

where $\text{Spec}(J)(R)$ is the $\mathcal{H}$-stratum of $\text{Spec}(R)$ corresponding to $J$. If $\mathcal{H}$ is an algebraic torus, then each $\mathcal{H}$-stratum is homeomorphic to the prime spectrum of a commutative Laurent polynomial algebra [22]. The $\mathcal{H}$-stratification theory is especially powerful when the number of $\mathcal{H}$-strata is known to be finite.

2.2 Effacement des dérivations

We follow the conventions of Cauchon [5]. Let $A$ be a unital $k$-algebra, let $A[X; \sigma, \delta]$ be an Ore extension of $A$, and let $S = \{X^n \mid n \in \mathbb{N}\}$. The deleting derivation homomorphism is a map $\phi : A \to A[X; \sigma, \delta]S^{-1}$ given by

$$a \mapsto \sum_{n=0}^{\infty} \frac{(1-q)^{-n}}{(n)_{q}!} \delta^n \sigma^{-n}(a)X^{-n},$$

where $(n)_{q}! = 1 + q + \cdots + q^{n-1}$.

Under appropriate restrictions, this sum is well-defined, defines an algebra map, and $\phi$ satisfies

$$X \phi(a) = \phi(\sigma(a))X \text{ for all } a \in A.$$

By the universal property of Ore extensions, we therefore have a unique map $\varphi$ extending $\phi$ and mapping $Y$ to $X$.

$$\begin{array}{ccc}
A[Y; \tau] & \xrightarrow{\varphi} & A[X; \sigma, \delta]S^{-1} \\
\downarrow i & & \\
A & \xrightarrow{\phi} & A[X; \sigma, \delta]S^{-1}
\end{array}$$

The image of $\varphi$ is the algebra obtained from the algorithm. Cauchon’s method is an process that starts with some iterated Ore extension $R^{(N+1)} = k[X_1][X_2; \sigma_2, \delta_2] \cdots [X_N; \sigma_N, \delta_N]$ and yields a sequence of algebras

$$R^{(j)} \cong k[Y_1] \cdots [Y_{j-1}; \sigma_{j-1}, \delta_{j-1}][Y_j; \tau_j] \cdots [Y_N; \tau_N]$$

terminating with $R^{(2)} = \overline{R} - \text{a quantum affine space}.$
2.3 Motivating example: quantum matrices

Let $k$ be an algebraically closed field, and assume $q \in k^\times$ is not a root of unity. The $k$-algebra of $n \times n$ quantum matrices $A$ is given by $n^2$ generators $X_{i,j}$ ($1 \leq i, j \leq n$) subject to relations: If \[
\begin{pmatrix} a & b \\ c & d \end{pmatrix}
\] is a $2 \times 2$ submatrix of $(X_{i,j})$, then

\[
ab = qba, \quad ac = qca, \\
bd = qdb, \quad cd = qdc, \\
bc = cb, \text{ and}
\]

\[
da = ad + (q - q^{-1})bc.
\]

Another realization of $A$ due to Faddeev, Reshetikhin, and Takhtadzhyan [9] is as the associative algebra over $k[q, q^{-1}]$ generated by formal entries $X_{ij}$ of a matrix $X$ such that

\[R(X \otimes I)(I \otimes X) = (I \otimes X)(X \otimes I)R\]

holds for the $R$-matrix

\[R = q^{-1} \sum_{1 \leq i \leq n} e_{ii} \otimes e_{ii} + \sum_{1 \leq i < j \leq n} e_{ii} \otimes e_{jj} + (q^{-1} - q) \sum_{1 \leq j < i \leq n} e_{ij} \otimes e_{ji}.
\]

There is a natural action of the algebraic torus $\mathcal{H} = (k^\times)^{2n}$ by $k$-algebra automorphisms on $A$ given by $(\alpha_1, \ldots, \alpha_n, \beta_1, \ldots, \beta_n) \cdot X_{i,j} = \alpha_i \beta_j X_{i,j}$. This induces an action on $\text{Spec}(A)$. It is well known that $A$ is an iterated Ore extension of $k$ and so is a noetherian domain. A result of Goodearl and Letzter [15] gives that all $\mathcal{H}$-primes of $A$ are completely prime and that $\mathcal{H}$-$\text{Spec}(A)$ is finite.

The deleting derivations algorithm gives rise to a natural embedding $\varphi : \text{Spec}(A) \to \text{Spec}(\overline{A})$, where $\overline{A}$ is given by the generators and relations of $A$, except that now the relation $da = ad + (q - q^{-1})bc$ is modified to $da = ad$. If we denote the generators of $\overline{A}$ by $\overline{X}_1, \ldots, \overline{X}_{n^2}$, then the $\mathcal{H}$-primes in the quantum affine space $\overline{A}$ have the form $Q_D = \overline{A}(\overline{X}_i \mid i \in D \subset \{1, \ldots, n^2\})$.

Cauchon deduced the image of $\varphi$ is a disjoint union of strata indexed by elementary combinatorial objects called Cauchon diagrams, which also parametrize $\mathcal{H}$-orbits of symplectic leaves in $M_n(k)$ [16] and restricted permutations, $\sigma \in S_{2n}$ such that $-n + i \leq \sigma(i) \leq n + i$ for $1 \leq i \leq 2n$ [2].

**Definition 2.1.** The Cauchon diagram of $J \in \mathcal{H}$-$\text{Spec}(A)$ is the unique set $D$ such that $\varphi(J) = Q_D$.

For $n \times n$ quantum matrices, the governing relation

\[da - ad = (q - q^{-1})bc\] (2.1)

implies that if $d$ is in a completely prime ideal, then either $b$ or $c$ must be as well. Thus, the Cauchon diagrams can be described combinatorially by $n \times n$ boxes such that if one square is filled, then every square to its left or every square above it is also filled. Explicitly, the general theory of deleting derivations yields the decomposition

\[\varphi(\text{Spec}(A)) = \bigsqcup_D \text{Spec}_D(\overline{A}).\]

Moreover, Cauchon proves [6] that the collections $\varphi^{-1}(\text{Spec}_D(\overline{A}))$ coincide with the Goodearl–Letzter stratification of $\text{Spec}(A)$. This gives an obvious direction for my future research.

As hinted in (2.1), the deleting derivations algorithm works well when the $\mathcal{H}$-primes of an algebra are completely prime, but this is often not the case. It seems, however, that I will be able to slightly modify Cauchon’s technique to analyze Manin’s super quantum matrices.
Problem 2.2. Using Cauchon’s deleting derivations algorithm, give a combinatorial parametrization of \( \text{Spec}(R) \) for super quantum matrices.

3 Results regarding \( \mathcal{H}\text{-Spec}(U^{\pm}[w]) \)

3.1 Formula for generators of \( U^{\pm}[w] \)

For \( 2 \times 2 \) quantum matrices the deleting derivations algorithm amounts to the transformation

\[
\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto \begin{pmatrix} a - qbcd^{-1} & b \\ c & d \end{pmatrix}
\]

We can express all of the generators of the resulting space as quantum minors or quotients of quantum minors; for instance, observe

\[
a - qbcd^{-1} = (ad - qbc)d^{-1} = \Delta_{12,12}\Delta_{2,2}^{-1}.
\]

Up to a scalar multiple, this result generalizes to \( U^{\pm}[w] \) for all reduced decompositions of \( w \).

Theorem 3.1 (G., Yakimov [17]). Let \( q_{\alpha} \in k^\times \) be the standard powers of \( q \). For all Weyl group elements \( w \) and reduced words \( i = (\alpha_1, \ldots, \alpha_l) \) for \( w \), the generators \( X_1, \ldots, X_l \) of the corresponding quantum affine space algebras are given by

\[
X_j = \begin{cases} (q_{\alpha_j}^{-1} - q_{\alpha_j})^{-1}\Delta_{1,\kappa(j)}^{-1}\Delta_{i,j}^{-1}, & \text{if } \kappa(j) \neq \infty \\ (q_{\alpha_j}^{-1} - q_{\alpha_j})^{-1}\Delta_{i,j}^{-1}, & \text{if } \kappa(j) = \infty \end{cases}
\]

This theorem allows one to bypass the intermediate steps in the Cauchon algorithm, and these \( q \)-commuting final generators link the deleting derivations algorithm to the initial cluster for the cluster structure of Geiß, Leclerc, and Schröer on \( U^{\pm}[w] \).

3.2 Unification of approaches to \( \mathcal{H}\text{-Spec}(U^{\pm}[w]) \)

Using representation theoretic techniques involving Demazure modules and quantized coordinate rings, Yakimov [32] constructed a family of graded, surjective, \( \mathcal{H} \)-equivariant maps \( \phi_w : R^w_0 \rightarrow U^-[w] \), where \( R^w_0 \) are subalgebras of \( R^w \) introduced by Joseph [19], and \( R^w \) are certain localizations of the positive part \( R^+ \) of the quantized coordinate ring \( R_q(G) \). For any element \( y \) in \( W^{\leq w} \), Yakimov proved the associated ideal \( I_w(y) \) — the image of a partially localized quantum Schubert cell ideal of \( R^+ \) under \( \phi_w \) — is a distinct, \( \mathcal{H} \)-invariant, completely prime ideal of \( U^-[w] \). Moreover, he showed that all \( \mathcal{H} \)-primes are of this form. Through this result he obtains a poset isomorphism

\[
W^{\leq w} \rightarrow \mathcal{H}\text{-Spec}(U^-[w]) \text{ given by } y \mapsto I_w(y).
\]

In 2004, Marsh and Rietsch defined a property of subwords of reduced expressions of Weyl group elements called left positivity and showed that for each \( y \in W^{\leq w} \) there is a unique left positive
subword whose index set $D$ satisfies $w^D = y$. Cauchon and Mériaux realized that the Cauchon diagrams (recall Definition 2.1) of the $H$-primes of $U^+[w]$ are precisely the index sets of all left positive subwords of $y$; this is the Cauchon–Mériaux classification, and from it they obtain a set bijection

$$W^{\leq w} \rightarrow H\text{-Spec}(U^+[w])$$

given by $w^D \mapsto J$ such that $\varphi(J) = Q_D$.

This gives an immediate problem, however, because Cauchon diagrams forget $H$-prime containment information. For example, in the very simple case of $2 \times 2$ quantum matrices, the following Cauchon diagram poset is not isomorphic to the Bruhat order interval $W^{\leq s_2s_1s_3s_2}$.

![Figure 1: Broken poset of Cauchon diagrams](image)

For arbitrary $U^\pm[w]$ it becomes a complicated matter to compare the defect in the diagram poset to the complete poset $H\text{-Spec}(U^+[w])$ because it is unclear which $H$-primes correspond to which diagrams in the general setting. This naturally gave rise to two questions of Cauchon and Mériaux [7, 5.3.2 and 5.3.3], which Yakimov and I answer; these seemingly different methods coincide when applied to $U^-[w]$.

**Theorem 3.2** (G., Yakimov [17]). *The Cauchon–Mériaux bijection and the Yakimov isomorphism of posets coincide.*

This solves the ideal containment problem of Cauchon and Mériaux and gives a nice quantum interpretation of the Bruhat order. Furthermore, our methods yield a new and independent proof of the Cauchon–Mériaux classification of $H\text{-Spec}(U^+[w])$. 
4 Present and Future Research

4.1 Generalized Deleting Derivations and Quasideterminants

Note that in (3.1) by utilizing the remaining $q$-commutation relations in the quantum affine space, we can rewrite the final generators in terms of quasideterminants. For example

$$a - qbcd^{-1} = a - bd^{-1}c = \left| \begin{array}{cc} a & b \\ c & d \end{array} \right|_{11}.$$ 

Cauchon [6] observed this phenomenon for $n \times n$ quantum matrices. Through a self-developed Mathematica package, I have generated much empirical evidence to suggest this result generalizes. I am currently extending it to all $U^\pm[w]$ algebras for any reduced decomposition of $w$.

**Conjecture 4.1.** For a fixed reduced decomposition of $w \in W$, the entire Cauchon algorithm applied to $U^\pm[w]$ is expressible in terms of a non-recursive quasideterminant formula. Such a formula would lead to interesting relations between quantum minors and quasideterminants. More importantly, however, it would provide an explicit method for analyzing $H$-primes in noncommutative algebras outside the scope of the deleting derivations algorithm.

4.2 KLR Algebras and Categorification of $U^\pm[w]$

This is joint work with Aaron Lauda, Greg Muller, and Ben Webster. For each quiver $\Gamma = (V,E)$, Khovanov, Lauda [20] and Rouquier [27] associate a family of graded $k$-algebras $R(\nu)$ over $\nu \in \mathbb{N}V$. The direct sum of Grothendieck groups of finitely generated projective graded $R(\nu)$-modules

$$\mathcal{A} = \bigoplus_{\nu \in \mathbb{N}V} K_0(R(\nu))$$

has the structure of a $\mathbb{Z}[q,q^{-1}]$-bialgebra with respect to induction and restriction, and there exists a canonical isomorphism from $\mathcal{A}$ to Lusztig’s integral form of $U_q^{-}(\Gamma)$

$$\psi : \mathcal{A} \to U_q^{-}(\Gamma),$$

which maps indecomposable projectives to the canonical basis [30]. The Serre relations then manifest themselves *categorically* as isomorphisms between direct sums of projective $R(\nu)$-modules. I will work towards solving the following:

**Question 4.2.** Is there an interesting categorification of the quantum Schubert cell algebras, and to what isomorphisms of modules do the relations in $U^\pm[w]$ lift?

The **Levendorskii–Soibelman straightening laws** give that $U^\pm[w]$ is an iterated Ore extension, and I will seek a categorical notion of this relation. Moreover, the algebras $U^\pm[w]$ do not depend on reduced decomposition of $w$, and choosing $w$ to be the longest Weyl group element $w_0$ yields $U^\pm[w_0] \cong U_q^\pm$. Thus, an interesting categorification might associate the Khovanov–Laua–Rouquier categorification of $U_q^\pm$ to any reduced decomposition of $w_0$. I believe that a strong combinatorial correspondence will exist between such a categorification and Mirković–Vilonen polytopes given their link to KLR algebras [28] and given the fact that every reduced decomposition of the longest word corresponds to a path from the *bottom* to the *top* of an MV polytope.
4.3 Superunital Cluster Domains

This project is joint work with Karl Mahlburg, Greg Muller, and Dylan Thurston. By the Laurent phenomenon \[10\], every cluster variable in a cluster algebra \(A\) can be written as a Laurent polynomial \(L\) in a fixed cluster \(X = \{x_1, \ldots, x_n\}\). We define the superunital domain \(S_A\) to be the subspace of \(\mathbb{R}_+^n\) defined by setting \(L(X) \geq 1\) for all cluster variables \(L\).

When the cluster algebra has finitely many cluster variables, that is, is of finite-type, evidence suggests \(S_A\) is a bounded topological polytope with a face poset dual to the subset of clusters poset. In this regard the faces of the superunital domain encode cluster combinatorics much like the generalized associahedron associated to the Dynkin diagram of the same type as the cluster algebra itself. In type \(A\) these are ordinary associahedra or Stasheff polytopes.

Each superunital domain supports a natural logarithmic volume form. When \(A\) is finite-type, then the integral of this volume form over \(S_A\) converges. Moreover, numerical evidence in all observed cases suggests the volume is always a rational multiple of the Riemann zeta function. For example when \(A\) is the cluster algebra associated to the \(A_3\) diagram we have

\[
\text{Vol}_{\log}(S_A) = \int_{S_A} \frac{dx_1 \wedge dx_2 \wedge dx_3}{x_1x_2x_3} = \zeta(3)
\]

Choosing a different starting orientation of the Dynkin diagram may lead to different cluster variables. Therefore, the corresponding superunital regions may slightly change. A fascinating occurrence, however, is that although the actual volume of \(S_A\) may change, the previously defined logarithmic volume will not.

We believe the logarithmic volume of the superunital region has much deeper ties to values of the zeta function. If \(A\) is a finite mutation type cluster algebra of rank \(n\) coming from a marked surface \(\Sigma\), then Teichmüller modular group \(\text{Mod}(A)\), a finite-index subgroup of \(\text{Mod}(\Sigma)\), acts on the cluster algebra by mapping seeds to seeds with the same quivers. Denote by \(S_A = S_A / \text{Mod}(A)\) the quotient of the superunital region under the induced action.

**Conjecture 4.3.** If \(A\) is a finite mutation type cluster algebra of rank \(n\) coming from a marked surface, then

\[
\int_{S_A} \frac{dX}{X} = q\zeta(n) \text{ for some } q \in \mathbb{Q}.
\]

4.4 Restricted GKZ Hypergeometric Systems and Gauss–Manin Connections

This project is joint work with Chris Bremer, J. William Hoffman, and Greg Muller. To any finite set \(A \subset \mathbb{Z}^n\), Gelfand, Kapranov, and Zelevinsky associate a holonomic system of differential equations whose solutions are generalizations of the classical Gauss hypergeometric functions \[12\]. The convex hull of the elements in \(A\) defines a polytope \(\Delta\). The following is well-known:

**Theorem 4.4.** Simply laced root systems and the \(B_2\) root system give rise to reflexive polytopes.

By a construction of Batyrev, this assignment yields mirror-symmetric pairs of Calabi–Yau hypersurfaces inside toric varieties \[1\]. Alternatively, there is a obvious Laurent polynomial one may assign to \(\Delta\) and this gives rise to an \((n+1)\)-parameter family of hypersurfaces inside the toric variety of \(\Delta\). Choosing the \(n\) root parameters (or non-constant Laurent polynomial coefficients) to be Weyl group invariant and rescaling them to \(-1\), we obtain a one-parameter family or pencil of hypersurfaces \(X_\lambda\). This procedure mirrors a construction of Verrill \[29\].
Example 4.5. If $\Pi$ is the $A_3$ root system, then the corresponding Laurent polynomial is

$$L_{A_3} = \lambda - xyz - \frac{1}{xyz} - xy - \frac{1}{xy} - x - yz - \frac{1}{yz} - y - \frac{1}{y} - z - \frac{1}{z}.$$ 

An interesting solution to the GKZ system with root system $\Pi$ is the generating function for the number of closed loops in the root lattice of $\Pi$ of length $n$. Alternatively, if $a_n$ is the constant coefficient of $(-L_\Pi + \lambda)^n$, then $\sum_i a_i \lambda^i$ is a solution to the GKZ system. By restricting the GKZ system to one parameter $\lambda$, we obtain recursive relations for the $a_i$. These identities were not previously known outside of the $A_n$ case, and in the modular setting it appears these numbers satisfy special $p$-adic congruences.

Now the collection $X = \{X_\lambda\}$ gives rise to a bundle with flat connection or local system over the space of parameters $U = \mathbb{C} \setminus \Sigma$ where $\Sigma$ is a finite set. Specifically, if $\pi : X \to U$ is a smooth map of smooth schemes over $\mathbb{C}$ with fibers $X_\lambda$, then (following Katz and Oda [21]) the de Rham cohomology sheaves $E_i = R^i \pi_*(\Omega^\bullet_X/U)$ give rise to the Gauss–Manin connection

$$\nabla : E_i \to \Omega^1_{U/\mathbb{C}} \otimes_{\mathcal{O}_U} E_i,$$

and its extension $E_i \to \Omega^2_{U/\mathbb{C}} \otimes_{\mathcal{O}_U} E_i$ is zero. This connection, which is difficult to compute, corresponds to an algebraic Picard–Fuchs differential equation. We explicitly find the families of Picard–Fuchs differential equations for all the collections of hypersurfaces $X_\lambda$ associated to root systems of rank less than five (except $F_4$), and we are working towards a general characterization.

References


