Trigonometric Identity

This manipulative illustrates a basic miracle of sinusoidal functions, namely: Any linear combination of the form

$$a\cos(\omega t) + b\sin(\omega t)$$

is itself "sinusoidal of circular frequency ω": it can be written in the form

$$A\cos(\omega t - \phi)$$

for some $A \geq 0$ and some ϕ.

(a). The green curve is supposed to represent the sum of the blue and yellow curves. To check that this is the case, run the cursor back and forth across the top window. Describe what you see, in words. Does this convince you that the green curve does represent this sum?

(b) Move the A slider and the ϕ slider. What happens to the red sinusoid when ϕ is *increased*? Is that consistent with the displayed formula $A\cos(\omega t - \phi)$?

(c) Be sure the a and b sliders are set at their initial positions, $a = 1$ and $b = 1$. Using the A and ϕ sliders, try to match the green curve with the red one. Record the best values of A and ϕ.

(d) It's useful to regard A and ϕ as the polar coordinates of a vector in the plane. To view this, press "A, ϕ" at lower right. Set A and ϕ so that this vector has rectangular coordinates $(2, -1)$. (Note: $\sqrt{5} \sim 2.236$ and $\arctan(0.5) \sim 0.467$ or about 27 degrees.) The next challenge is to adjust the a and b sliders so that the green curve matches the red one. This may be much more counterintuitive than the matching exercise in **(c)**. Record your best values for a and b.

(e) Now reset a and b to zero. You will do the same estimation as in **(d)** again, but this time taking advantage of a display of the values of a and b. For this, press "All." Now see how close you can come to matching the red curve with the green one. Record your best values of a and b.

(f) This is all evidence for a trigonometric identity of the form

$$a\cos(\omega t) + b\sin(\omega t) = A\cos(\omega t - \phi).$$

Make a clear statement, perhaps using a graph like the righthand window in the manipulative, of the relationship between A, ϕ, a, and b.