Questions for use with "Phase Lines"

1. Don't touch anything, for the moment; just describe what you see when the manipulative opens.

Once you have done that, here are some more detailed questions for you.

(a) I claim that the direction field you see before you represents an autonomous ODE. What feature (or features) of the direction field is (or are) relevant to this claim? What features are not relevant?

(b) Can you find the differential equation on the screen? There is a parameter, or constant, called a in the equation. You will notice a slider which is now set so $a = 0$. With $a = 0$, this is called the *logistic* equation, and it represents population growth (or decay) in the presence of a limiting population. Please explain the form of the equation as well as you can, when you think of it as modeling this biological system. A more general form of the logistic equation is

$$y' = ry(b - y).$$

Can you explain the significance of the parameters r and b?

(c) What does the red line indicate? How about the green line?

(d) When you move the cursor across the graph window, you change the recorded value of y_0. What do you suppose the significance of y_0 is?

2. Now we'll begin to explore the manipulative.

(a) Click the mouse key while the cursor is positioned somewhere on the graph window. A curve forms. What is it about the curve that supports the belief that it represents a solution to the ODE? Click at several more positions, letting the curve complete after each click. What is the relation between where you have your cursor and where the solution curve begins? Does the horizontal component of the position of the cursor have any significance?

(b) Now click on the "Phase Line" button. A new, very narrow, window opens up. Position the cursor between the lines in the graph window. Describe what appears in the Phase Line window, and explain what its significance is. What happens when you move the cursor around?

(c) Now click somewhere on the graph plane and describe the effect.

(d) Finally, position the cursor to represent the smallest positive value of y_0 that you can and click to generate a solution curve. Estimate the value of y at $t = 8$.

3. Now we'll disucss the parameter a.

(a) Explain, as well as you can, why this a represents the harvest rate, and what it means to say this.

(b) Position the cursor over the slider labeled a, depress the mouse key, and move the mouse. Describe what happens to the graphing plane.

(c) Set $a = 0.20$ and describe what has happened to the critical points. Can you explain this behavior in terms of the population model? What does the region between the red line and the t-axis represent? How about between the red and green lines? Above the green line?

(d) Now position the cursor to represent the smallest value of y_0 that you can which is above the red line, and click to generate a solution curve. Estimate the value of y at $t = 8$. How does this compare with your answer to **2(d)**?

4. To understand what you discovered in **3(d)**, click on the button labeled "DE Graph."

(a) Describe the significance of the contents of the new window. What is the significance of the red annulus and the green disk? (I am not asking why one is an annulus and the other is a disk—I don't know the answer to that myself!)

(b) Move the a slider back to $a = 0$. What happens to the curve in the DE Graph window? Compare the maximum value of the graph displayed there now to what it was when $a = 0.20$. Can you relate this to the different answers to **2(d)** and **3(d)**?

(c) Now continue to change the value of a till the red and green spots collide, and then beyond. Describe what happens to the solution curves as the parameter a makes this transition.

5. Now click on the "Bifurcation plane" button. You see a parabola in the window. Move the a cursor back to the left and watch what happens.

(a) What direction should the arrows be pointing when $a = 0.25$?

(b) What is the equation of the parabola in the bifurcation plane?

6. There is a lot more to explore in this manipulative. Play around with it and describe what you find as you do. All comments are welcome!

7. Do you have any comments about this manipulative and this accompanying guide?

(a) Are there some point that are more obscure than others on the screen?

(b) Can you think of ways we might learn from your interaction with this manipulative which we have not exploited today?