

## Questions for use with “Fourier Coefficients”

1. Don't touch anything, for the moment; just describe what you see when the manipulative opens. Press the “Formula” key to see more detail.

Once you have done that, here are some more detailed questions for you.

(a) Leave the  $b_1$  slider alone for the moment, but move the  $b_2$  slider and watch what happens. Leave it at some nonzero value and move the  $b_1$  slider. While you are holding the mouse key down, a pale white curve shows up in the graphing window. What is it? What is the yellow curve?

(Attention: Note the various ranges of the sliders. This reflects the fact that for most functions the size of the Fourier coefficients  $b_k$  and  $a_k$  fall off as  $k$  gets large.)

(b) Now press “Cosine Series” and describe what happens. Select a value for  $a_0$ . The line is at height  $a_0/2$ . Is that a bug? Then move  $a_1$  and describe the effect.

You can play around with these and make graphs with interesting shapes. Remember, like sines and cosines, the functions you are making are periodic, of period  $2\pi$ . We are seeing only the stretch between  $-\pi$  and  $\pi$ .

2. Now press the key labeled A. The challenge is to match this curve as well as you can with part of a Fourier series.

(a) Is the green function even or odd? Should we use sines or cosines to approximate it? Make a selection and make some headway.

(b) Cosines are odd, sines are even. Try to express, verbally, the symmetry properties exhibited by even sines and by odd sines. This particular function is a sum of one or the other, even or odd, sines. Which?

(c) Select the appropriate parity and continue to approximate the function. Jot down the values of  $b_k$  when you are satisfied.

3. There is a precise sense in which the Fourier coefficients are the *best* multiples of sines and cosines one can possibly use in a sum approximating the function. This is essentially the “least squares fit,” but we don't need to get into the theory of this right now. The first thing to know is that there is a single number that measures how near together two functions are. It's a kind of distance between them.

To see the distance between a green target and the yellow Fourier approximation, press “Distance.”

(a) Now start to adjust the coefficients to make the distance smaller. The next thing to know about this distance is that in its variation with each of the Fourier coefficients, keeping all the others unchanged, it has exactly one minimum value. Adjust the sliders in sequence so each gives minimum distance. Jot down the coefficients, and the minimal

distance. Does the approximation look better than you were able to eyeball? I hope this helps convince you that the “distance” is a useful thing.

**(b)** Now push “Reset.” We’re going to minimize distance again, but this time adjust the sliders in a different sequence—maybe from bottom up. When you are done with this, compare the optimal values of the coefficients with what you had in **3 (a)**.

I think they should have come out exactly the same (with perhaps a slight variation in the last digits, due to the fact that you were constrained in what exactly you were allowed to set the coefficients at). Do you find this surprising? You can think of the Fourier coefficients as the best multiple of each sine and cosine, and “best” here is completely independent of what multiples of the others you use. The word for this effect is *orthogonality*.

By the way, would you care to speculate about what the exact values of the Fourier coefficients are in this case, based on what you have measured? The function has values  $\pm\pi/4$ ; you could compute them if you like.

**(c)** Finally, push “Cosine Series” and check what settings of the cosine coefficients minimize distance. Is this good?

**4. (a)** Now press “Distance,” “Reset,” and “F,” and go through the same process with target F: decide whether it requires cosines or sines, and whether it needs all of them, only the even ones, or only the odd ones (and try to enunciate the difference in the symmetry properties between even cosines and odd cosines—it’s different than in the even/odd sines case); eyeball your way to an approximation; press “Distance” and adjust your answers. You might want to check the amazing orthogonality property again.

**(b)** Compare the minimum distances achieved for A and for F. Does the approximation in fact *look* better for F than it did for A? What feature of the target graphs do you think might correlate with this difference? You might want to check out the other target functions in this manipulative to get more evidence. Remember, these are portions of *periodic* functions, so D isn’t as nice as it looks.

**5.** Use this tool to discover the first few Fourier coefficients of function C. Compare C to F, and compare the coefficients. Can you explain the coincidence? It will help if you write down starts of the two Fourier series.

**6.** Do you have any comments about this manipulative and this accompanying guide? Are there some point that are more obscure than others?