

Complex Exponential

This manipulative illustrates the Euler formula

$$e^{a+bi} = e^a(\cos(b) + i \sin(b)).$$

(a) When the tool opens, the cyan point in the left window is at $z = 1 + \pi i$. Calculate the exponential of this complex number. The cyan point in the right window is the exponential of the cyan point in the left window. The readout under the right hand screen lists the value of the cyan point there as 1.65. What is the exact value? Is this a good estimate?

(b) Move the t slider at lower right. This controls the position of a yellow point in the right window, moving along a straight line between 0 and the cyan point (which is now at $(1/2) + \pi i$, but let's do this for the general case, when it is at $a + bi$). Write down an expression for the position of the yellow point in the left window in terms of t .

(c) The t slider also controls a yellow point in the right window. The complex exponential carries you from the left window to the right window. What is the position of the yellow point in the right window, in terms of a , b , and t ?

Lesson: a straight line through the origin gets carried by the complex exponential to a spiral. Move the blue point in the left window around using the mouse button, and watch the effect.

(d) Where would you position the cyan dot in order to produce a circle on the right hand screen? Make a statement about this, as in “the complex exponential carries [some line] to a circle.” By the way, could that be *any* circle?

(e) Now set $b = 4$, using the slider at bottom right. Then grab the a slider and move it back and forth. The cyan line moves back and forth along a dark blue line in the left window. Describe this line: it's the set of complex numbers such that what? Meanwhile, a dark blue line appears on the right hand screen as well. This is where the complex exponential carries the left hand blue line. Write an expression for a general point on the left line and compute its exponential. Does the picture reflect what you computed?

(f) Now set $a = 1/2$ and move the b slider around. Carry out exercise (e) again.