

DANIEL M. KAN (1927–2013)

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MOERDIJK

1. LIFE

In Holland. Daniel (or Daan in Dutch) Marinus Kan was born on August 4, 1927, and grew up as the only child in a liberal Jewish family in the southern quarter of Amsterdam, where his father worked as a lawyer. In 1939 he entered the Barlaeus Gymnasium, a secondary school in the center of Amsterdam, then and to this day one of the best schools in Amsterdam. He was only able to stay there for two years, because after the school year 1940/41, under German occupation, Jewish children were no longer allowed at that school. Instead, he went to the Jewish Lyceum (*Joods lyceum*). In the summer of 1943 a very difficult time for Kan and his family began. Together with his parents, he was picked up and sent to Westerbork, a transition camp near the eastern Dutch border. They stayed there for half a year, and were next sent to Bergen-Belsen where they remained for fifteen months. Both of his parents died of typhus soon after liberation. Dan himself barely survived, and he stayed in Germany for another three months to recover.

In the summer of 1945 Kan returned to Amsterdam, where he entered the last year of high school at the Barlaeus Gymnasium. He was interested in mathematics, but the prospect of becoming a school teacher or having a job in an insurance firm did not appeal to him. It was the Dutch topologist L.E.J. Brouwer who pointed out other career options to him that a degree in mathematics would offer. Consequently, Kan started his university studies in 1946 at the University of Amsterdam, in the program Mathematics and Physics with Chemistry. In 1948 Kan received his first degree (*kandidaatsexamen*), and in 1950 he completed his studies (*doctoraal examen*, comparable to a Masters degree).

Two events from Kan's years at the University of Amsterdam turned out to have a decisive impact on some of his later choices. First of all, Kan always spoke with admiration about the lectures by Professor Johannes De Groot about "differentiation and integration." In the Spring of 1949 De Groot gave a seminar lecture about the book by

the Princeton mathematician Solomon Lefschetz about Topology, and he organized a small reading group at his home to work through this book. Kan joined this reading group, together with several other young Dutch mathematicians, among them the late T.A. Springer. Secondly, after his *kandidaats* degree, Brouwer asked Kan to become his assistant. Brouwer left Kan a lot of freedom, and Kan spoke of this assistantship as very inspiring. In this way, both Brouwer and De Groot influenced Kan's future career to a high degree.

In Israel. In February of 1951, equipped with letters of recommendation from De Groot and Brouwer, Kan left for Israel. After a few weeks there, he managed to obtain a job at the Weizmann Institute in a seismic oil exploration project. Kan had to do the mathematical calculations that this search required. (Oil was never found.) Kan described this work as rather monotonous and not very inspiring. After a year, Kan had to serve in the army. But the army allowed him to do his service at the Weizmann Institute itself, and he could thus stay there for another two and a half years. His job offered him a lot of spare time, and he began to think about topology again. In the Spring of 1954, Samuel Eilenberg came from Columbia University on a visit to the Hebrew University in Jerusalem. (At the time, Eilenberg was already one of the leading figures in algebraic topology, due to his work with Saunders Mac Lane and the influential *Foundations of Algebraic Topology* which he had just written with Norman Steenrod.) Kan knocked Eilenberg's hotel room door, and explained his simplicial description of homotopy groups. Eilenberg asked him if he could prove the homotopy addition theorem, and Kan returned a week later with a proof. Eilenberg told Kan that he had a thesis there, engineered an ad hoc arrangement giving Kan the status of graduate student at the Hebrew University, and in the summer of 1954 Kan submitted his thesis. He formally received his PhD in 1955.

In the meantime Kan had married Nora Poliakof, the daughter of a Dutch general practitioner in Amsterdam, and like Kan survivor of Bergen-Belsen. Nora too had lost both parents during the war, and immediately after had emigrated to Israel. Dan and Nora had four children, Ittai (1956), Michael (1957), Tamara (1962) and Jonathan (1965). Jonathan died in 1973 of leukemia, a heavy blow for the Kan family. Nora died in August 2007.

In the United States. After his PhD, Kan took up a postdoctoral position for a year (1955-56) with Eilenberg at Columbia University. During that year, he wrote three groundbreaking papers about simplicial sets, and two about adjoint functors. Subsequently, Kan went

to Princeton for a year, before returning to Israel to take up what we would now call a tenure track position at the Hebrew University. However, he returned to the USA in 1959 to take up an Assistant Professorship at MIT. After just four years he was promoted to Full Professor there. Kan stayed at MIT for the rest of his academic career, until his formal retirement in 1993. After his departure, Kan did not have much contact with Holland anymore. He had lost his family and some of his best friend from his youth. (His wife Nora did go back regularly, to visit some of her school friends.) But Kan was proud of his membership of the Dutch Royal Academy, and published regularly in its mathematics journal, *Indagationes Mathematicae*.

Dan was an enthusiastic cyclist. Up to very late in his life, dressed in a striking red outfit, he would climb on his fancy mountain bike, to go and have a cup of coffee with one of his colleagues in the neighborhood, to work on a joint paper or simply to share the latest mathematical gossip and give some piece of sharp advice. Ismar Volic has provided a vivid image of the important role Dan played in his mathematical community.

Dan Kan died at home on his 86th birthday, on August 4, 2013, following a brief illness.

2. THE KAN SEMINAR.

Kan had very strong views about his role as a teacher. He had a modern view of the teaching profession, having little use for the convention of a lecture. He gave very few lecture courses at MIT, and rarely spoke at conferences. Instead, he devised various alternatives. He introduced an undergraduate seminar in basic algebraic topology, in which students were given a list of definitions and theorems and were challenged to provide examples and proofs and present them in class.

In 1969 Kan instituted the “Kan Seminar,” a literature seminar devoted to classic papers in algebraic topology. The charter class included Tadatoshi Akiba (Mayor of Hiroshima from 1999 to 2010), Ken Brown (now at Cornell), Dan Burns (University of Michigan), Hans Salomonson (Aarhus), Bruce Williams (Notre Dame), and W. Stephen Wilson (Johns Hopkins). It has run every fall since then (with perhaps one exception). While the list of papers has evolved somewhat, the course has remained very close to Dan’s original conception. One of Dan’s principles was that this was not a recruiting device to attract PhD students. The seminar was held in Dan’s office, which featured a long couch and a prominent copy of C. Allan Gilbert’s print “All is Vanity.” It met very early—8:00 AM—and he would hold office hours before it.

A central objective was the formation of a cohort of graduate students who knew each other well and worked together. To further that goal, every fall he and Nora hosted a party at his house and invited some 35 Boston area mathematicians along with the seminar participants. They may not have realized it, but he thought of this as an important part of the seminar, and it stopped when he stopped leading the seminar. The Kan seminar has spawned a variety of similar seminars around the world, notably Emily Riehl’s recent “Kan extension seminar” [Emily Riehl, The Kan Extension Seminar: An Experimental Online Graduate Reading Course, *Notices of the AMS* 61 (2014) 1357–1358].

3. WORK

Kan struck an independent course from the very start. Eilenberg had initiated the use of singular simplices in 1944 with the objective of constructing a transparently functorial homology theory, and Serre used a cubical analogue to ease his work on the homological structure of fibrations. Kan initially preferred cubical sets over simplicial sets. He realized that in fact one could define the homotopy groups of a cubical set provided one could fill in the boundary of a cube minus one face: this was the “extension” or “Kan” condition. He was persuaded of the efficacy of simplicial objects in place of cubical objects by John Moore’s proof that a simplicial group automatically satisfied the extension condition; and starting with this third paper everything is written simplicially. Very quickly he subsumed a lot of classical homotopy theory under the simplicial tent. He constructed a model for Serre’s loop space using simplicial groups which were free on generators preserved by degeneracies. These became one of Kan’s favorite ways of thinking about a homotopy type. The Hurewicz map was simply abelianization, and a simple proof of the Hurewicz theorem emerged. For connected simplicial sets one could dispense with inverses; this is the “Dold-Kan” theorem. This line of research culminated in Kan’s demonstration (1959) that these free simplicial groups are just as efficient as CW complexes in their representation of homotopy types. This perspective contributed to the Kan-Thurston theorem (1976) asserting that any connected space is homologically equivalent to the classifying space of a discrete group.

The passage back and forth between spaces and simplicial sets had interesting formal properties, which Kan codified in the language of adjoint functors. (The name was insisted upon by Eilenberg.) This formalism has become so ingrained in mathematics that it seems incredible that it had an author. In the same paper, Kan defines limits,

colimits, and what we know today as Kan extensions. (Mac Lane’s dictum “all concepts are Kan extensions” is only a mild exaggeration.)

Kan played an important early role in the development of stable homotopy theory. Spectra were first described by Lima in 1959, and by 1963 Kan had published his account of “simplicial spectra,” simple combinatorial objects behaving like simplicial sets with negative dimensional simplices. Kan’s colleague George Whitehead showed (1962) that homology theories could be represented by spectra, and the two of them collaborated on the development of the theory of simplicial spectra. They investigated the smash product of spectra, and orientability with respect to a sequence of ring spectra obtained from the group spectrum model of the sphere spectrum by dividing by terms in the lower central series.

Kan’s interpretation of the Hurewicz theorem had already suggested that the lower central series would be interesting. He had given an interpretation of J. H. C. Whitehead’s “certain exact sequence” in these terms. He helped Raoul Bott’s student Edward Curtis come to and prove a connectivity theorem for terms in the mod p lower central series, a result underlying many convergence results to this day. Soon the “Six Author paper” (A. K. Bousfield, E. B. Curtis, D. M. Kan, D. G. Quillen, D. L. Rector, and J. W. Schlesinger, 1966) used the mod p lower central series to produce a spectral sequence coinciding stably with the Adams spectral sequence, with an explicit E_1 term (described in terms of the “Lambda algebra”). Kan’s student Stewart Priddy went on to make the unstable picture explicit, and to found the “Koszul duality” movement.

Also at this time Kan was serving as guide to Daniel Quillen as Quillen developed the axiomatic framework of model categories. Quillen’s book *Homotopical Algebra* incorporates much of Kan’s world view, and Kan adopted the framework of model categories in his later work.

From this point on much of Kan’s research was conducted in collaboration with former students. With A. K. Bousfield, he gave two other constructions of the unstable Adams spectral sequence. The second of these was based on an exhaustive study of cosimplicial spaces and the tot construction on them, and appears in the book *Homotopy Limits, Completions and Localizations*, which Kan always referred to as “the Yellow Monster.” This served as a textbook for a generation of topologists learning simplicial methods, and provided a general foundation for the process of localization and completion explored earlier by Artin and Mazur, Sullivan, and Quillen. It made extensive use of the notion of a nilpotent space, due to Kan’s student Emmanuel Dror Farjoun, an optimal weakening of simple connectedness or simplicity. Bousfield

and Kan published a series of papers filling out various aspects of this work.

In the late 1970's Kan began a long collaboration with Bill Dwyer, who had received his PhD under Kan's direction in 1973. In 1980 they linked Quillen's theory of model categories to another conception of what a homotopy theory is by defining a simplicial enrichment of a model category, using a device they termed "hammock localization." Over the next twenty years Dwyer and Kan explored a wide swath of homotopy theory. They (along with Kan's student Chris Stover and others) developed a strategy for classifying topological realizations of various kinds of homotopical data, and investigated the accompanying obstruction theory. They identified the "centric" condition on a diagram, a condition that is satisfied quite often and that dramatically simplifies the obstruction theory for realization. This has been fundamental in developments ranging from p -compact groups to elliptic cohomology. With Mike Hopkins and Jeff Smith, Dwyer and Kan explored the homotopy theory of cyclic sets.

Much of Kan's effort in the late 1990's and early 2000's was focused on a reconceptualization of homotopy theory, trying to understand what a homotopy theory is rather than understanding homotopy-theoretic properties of spaces (or simplicial sets). The fruits of this effort are presented in "the Blue Beast," *Homotopy limit functors on model categories and model categories*, by Dwyer, Kan, Philip Hirschhorn, and Jeff Smith. Quillen had constructed the homotopy category by inverting weak equivalences; and Dwyer and Kan had shown that the entire homotopy theory is captured by the weak equivalences. The Blue Beast provides an axiomatic development of homotopy theory from that perspective; the other components of a model category—the cofibrations and fibrations—assume their proper role as computational conveniences. This perspective was carried further, into the realm of higher category theory, in Kan's final collaboration, with Clark Barwick.

Dan Kan had a very strong need to express his mathematics in the most transparent way possible. Things had to fit together tightly for him; he often said, "If it rattles, it's not quite right." This imperative gives his articles a certain timelessness.

4. ISMAR VOLIC, WELLESLEY COLLEGE

Soon after I arrived at Wellesley College in 2006, Dan knocked on my door. He had heard from my colleague and his former student Phil Hirschhorn that I had moved to the area and happened to be

near my apartment on one of his legendary daily bike rides. Our first conversation over what would become the requisite cup of coffee must have gone well; Dan became a regular visitor and I considered it a great professional accomplishment that one of my mathematical heroes had added me to his “topological bike tour.” He always came unannounced, a spontaneity I came to cherish as his visits made me put the grind of ordinary days on pause for an hour or so.

Naturally, Dan and I quickly found a common language in mathematics, but we especially bonded over the shared experience of war in our youth (albeit half a century apart) and, more importantly, over a common sensibility to what a balanced life should look like. He had modeled his life on the simple hierarchy where family and friends stood well above everything else, and I have tried to emulate his example by arranging my schedule so that it allowed plenty of time for both. I will miss Dan’s invaluable help in maintaining sight of the things that matter.

Dan was wonderful to my children as well, always bringing them thoughtful, simple gifts that fostered imagination and playfulness. They came to regard him as something of a grandfather and will miss him as much as my wife and I will. Fortunately we inherited Dan’s trove of puzzles, brain teasers, and games and look forward to many hours of fun and play – exactly what Dan would want us to do in his memory.

5. STUDENTS

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|-----------------------|------|
| James Schlesinger | 1964 |
| John Dennett | 1965 |
| Aldridge Bousfield | 1966 |
| Robert Knighten | 1966 |
| David Rector | 1966 |
| Stewart Priddy | 1968 |
| Robert Walker | 1968 |
| Emmanuel Dror Farjoun | 1971 |
| William Dwyer | 1973 |
| Jerrold Grossman | 1974 |
| Philip Hirschhorn | 1977 |
| Javier Bracho | 1981 |
| Jeffrey Smith | 1981 |
| David Blanc | 1988 |
| Christopher Stover | 1988 |