1. Let $G_{n,k}$ be the graph with vertex set $V = \{0, 1, 2, \ldots, n - 1\}$ considered cyclically, and an edge between $i$ and $j \neq i$ if and only if $i - j \pmod{n} \leq k$ (e.g. in $G_{10,2}$ vertex 1 is adjacent to vertices 10, 0, 2, 3.). $G_{n,k}$ is a $2k$-regular graph (every vertex has degree $k$).

(a) What are the eigenvalues of the Laplacian of $G_{n,k}$?

(b) For $k$ fixed, how does $\lambda_2$ behave as a function of $n$ (just give the leading term as a function of $n$ in $O(\cdot)$ or $\Omega(\cdot)$ notation)? Does the family of graphs $G_{n,k}$ (parametrized by $n$) constitute an expander family?

(c) What is the number of spanning trees of $G_{n,2}$? There is a simple closed-form formula for it. (To guess it, you may want to experiment with matlab for a few values of $n$. To prove it, you may want to try to derive a recurrence relation.)