We have shown that any solution to Kirchhoff’s laws and boundary values is unique, and defined such a solution in terms of probabilities. Now we define one in terms of the number of spanning trees in the graph.

A **tree** is a graph in which there is exactly one simple path between any two vertices, i.e. a connected graph with no cycles.

A **spanning tree** of a graph \( G = (V, E) \) is a subgraph of \( G \) that is a tree and contains all of \( V \). A graph may have many spanning trees.

Given an electrical network \( G = (V, E) \) with all conductances = 1 (result generalizes to arbitrary conductances) and \( s, t \in V \).

Let \( N(s, a, b, t) = \) the number of spanning trees with a path from \( s \) to \( t \) that traverses edge \( (a, b) \) from \( a \) to \( b \) and \( N \) be the total number of spanning trees of \( G \).

**Thm:** The current \( i_{ab} = \frac{1}{N}(N(s, a, b, t) - N(s, b, a, t)) \forall (a, b) \in E \) defines a unit flow from \( s \) to \( t \) that satisfies Kirchhoff’s laws.

Note that this is like \( i_{ab} = Pr_T \{ T \text{ has a path from } s \text{ to } t \text{ traversing } (a, b) \text{ from } a \text{ to } b \} - Pr_T \{ T \text{ has a path from } s \text{ to } t \text{ traversing } (a, b) \text{ from } b \text{ to } a \} \).

The number of spanning trees relating to current then has implications for uniformly generating a random spanning tree from a graph. The probability over all spanning trees of an edge \( (a, b) \) being in a tree is \( i_{ab} \). So include an \( (a, b) \) in the spanning tree with probability \( i_{ab} \). Once a decision has been made for an edge, if it is not put in the tree, it can be deleted from the graph and a spanning tree generated for the remaining graph. If it is put in the tree, \( a \) and \( b \) can be combined into one node and a spanning tree generated for this new graph.