1. Compute the length of the \( \mathbb{C}[x, y] \)-module
   \[ M = \mathbb{C}[x, y]/(y^2 - x^3, x^2 - y^3). \]
   Find the maximal ideals associated to \( M \), and for each of them find the length of \( M_m \).

2. Find the Hilbert-Samuel series of \( \mathbb{C}[x, y] \) with respect to the ideal \((x^2, y^3)\).

3. Let \( I \) be the ideal \((x_1, \ldots, x_n) \subset \mathbb{C}[x_1, \ldots, x_n] \). Describe this ideal as a module over \( \mathbb{C}[x_1, \ldots, x_n] \) by generators and relations.

4. Let \( I \) be the ideal in \( \mathbb{C}[x, y] \) consisting of polynomials which vanish when \( y = 0 \) and also at the points \((0, 1), (0, -1)\). Find a finite set of generators for \( I \).

5. Let \( I \subset R \) be a proper ideal, and \( G^*R \) be the associated graded ring with respect to the \( I \)-adic filtration. Suppose \( \cap_{n \geq 0} I^n = 0 \). Show that if \( G^*R = R/I \oplus I/I^2 \oplus \ldots \) is a domain then \( R \) is a domain. Is the converse true?

6. Let \( I \) be the intersection of finitely many maximal ideals in \( \mathbb{C}[x, y] \). Show that \( I \) can be generated by two elements.

7. (a) The group \( \mu_n \) of roots of unity of order \( n \geq 2 \) acts on \( \mathbb{C}[x, y] \) by \( a(x, y) = (ax, a^{-1}y) \). Describe the algebra of invariants \( \mathbb{C}[x, y]^{\mu_n} \) by generators and relations (you should need three generators).
   (b) Let \( G \) be the group generated by \( \mu_n \) and the element \( g \) sending \((x, y)\) to \((y, -x)\). Describe the algebra of invariants \( \mathbb{C}[x, y]^G \) by generators and relations (again you should need three generators).

8. Which of the following rings are Noetherian:
   (i) \( \mathbb{Z}[x_1, x_2, \ldots]/(x_{i+1} - ix_i, i \geq 1) \)
   (ii) \( \mathbb{C}[[x, y]]/(x^2 - y^3) \)
   (iii) The ring \( R_\infty \) of holomorphic functions on \( \mathbb{C} \).
   (iv) The ring \( R_r \) of holomorphic functions on the open disk of radius \( r \).
   (v) The ring \( R_0 = \cup_{r > 0} R_r \) of germs of holomorphic functions near 0 (nested union).

9. Let \( R \subset \mathbb{C}[x] \) be the ring of polynomials such that \( f(j) = f(-j) \) for \( j = 1, \ldots, m \). Show that \( R \) is generated by 2 elements, and find the defining relation between these elements. Is \( R \) normal?

10. (a) Let \( G \) be a finite subgroup of \( GL_n(\mathbb{C}) \). Show that the algebra \( \mathbb{C}[x_1, \ldots, x_n]^G \) is normal.
    (b) Show that the algebra \( R = \mathbb{C}[x, y, z]/(z^2 - xy) \) is normal (you may use problem 7).