1. Let q be a prime, $R = \mathbb{Z}$ and $S = \mathbb{Z}[\sqrt{q}]$. Find the primes in S lying over each prime $p \in \mathbb{Z}$ (use quadratic reciprocity).

2. Let $R = \mathbb{C}[x, y, z]/(x^2 + y^2 - z^2 + 1)$. For a nonzero $v \in \mathbb{C}^3$, let $E_v \subset (\mathbb{C}^3)^*$ be the space of linear functions a such $a(v) = 0$, and let a_1, a_2 be a basis of E_v. Find all v for which R is module finite over $\mathbb{C}[a_1, a_2]$ (i.e., we have an instance of Noether normalization).

3. (a) Let I be the ideal of polynomials in $\mathbb{C}[x, y, z]$ which vanish on the three coordinate axes. Find a finite set of generators for I.

(b) Let J be the ideal of polynomials in x, y, z that vanish on the plane $z = 0$ and on the z-axis. Find a finite set of generators for J.

(c) Let $R = \mathbb{C}[x, y, z]/J$. Find the Krull dimension of R.

(d) Let S be a ring of Krull dimension d, and $\mathfrak{p}_0 \subset \mathfrak{p}_1 \subset ... \subset \mathfrak{p}_r$ be a strictly increasing chain of primes in S which cannot be refined (i.e., we cannot insert more primes into it). Does it follow that $r = d$?

4. Let I be the ideal in $\mathbb{C}[x, y]$ generated by xy^2 and x^2y. Construct infinitely many primary decompositions of I.

5. Let p be a prime. Show that there are exactly four isomorphism classes of Artinian rings with p^2 elements, and describe these rings. Which of them are algebras over the field $\mathbb{F}_p$?