1. Let $R = \mathbb{C}[x]$.

(a) Describe all saturated multiplicative subsets S in R.

(b) For each such S, describe the localization $S^{-1}R$.

(c) For which S is $S^{-1}R$ a finitely generated $\mathbb{C}$-algebra? A local algebra?

(d) What are the S-saturated ideals of R (for each S)? What is the saturation of the non-saturated ones?

(e) For any finitely generated module M over R and any S, describe $S^{-1}M$.

2. Let $R = \mathbb{C}[x, y]/(y^2 - x(x-1)(x-2))$.

(a) Describe $X = \mathrm{Spec}R$ (and draw a picture of the real points).

(b) Describe the open sets in X in Zariski topology.

(c) Let P be any point on the curve $y^2 = x(x-1)(x-2)$, and M be the ideal consisting of $f \in R$ such that $f(P) = 0$. Show that M is a projective (=locally free) R-module of rank 1, but it is not free.

(d) (harder!) Which of the open sets on X are principal? For which n is there a set of distinct points $P_1, ..., P_n$ on the curve $y^2 = x(x-1)(x-2)$ such that $X \setminus \{P_1, ..., P_n\}$ is not principal?

Hint. You may need to use the addition law on the elliptic curve.

3. Let $R = \mathbb{C}[x_1, ..., x_n]$, and $\phi : R^m \to R^n$ is a module map, given by a matrix $[\phi] = (a_{ij})$, $a_{ij} \in R$. Let $M = \mathrm{Coker}\phi$. Describe the support of M explicitly (as $V(\mathfrak{a})$, where the ideal $\mathfrak{a}$ is defined by polynomial equations). Consider first $m < n$, then $m = n$, then $m > n$.