2. Let $\mathcal{F} = \oplus \mathcal{F}^{(m)}$ be the semiinfinite wedge space.
   (a) Show that $\mathcal{F}$ is an irreducible representation of the Clifford algebra generated by $\hat{v}_j, \check{v}_j$.
   (b) Compute $Tr_{\mathcal{F}}(q^d z^m)$, where $d$ is the operator multiplying homogeneous elements by their degree, defined by $deg(\psi_0) = 0$, $deg(\hat{v}_j) = j$, $deg(\check{v}_j) = -j$, and $m$ is the operator which acts by multiplication by the number $m$ on $\mathcal{F}^{(m)}$.
3. Using the fact that $\mathcal{F} = \mathcal{B}$, compute the answer to problem 2 using the bosonic realization. Deduce that
   \[
   \prod_{n \geq 0} (1 - q^n z)(1 - q^{n+1} z^{-1})(1 - q^{n+1}) = \sum_{m \in \mathbb{Z}} (-z)^m q^{m(m-1)/2}
   \] (Jacobi triple product identity). Substitute $z = q^{1/3}$ and obtain Euler’s pentagonal identity for $\prod_{n \geq 1} (1 - p^n)$.
4. Let $\hat{\mathfrak{g}}$ be the affine Lie algebra associated to a simple Lie algebra $\mathfrak{g}$. Let $a \in \mathfrak{g}$ and $a(z) = \sum a[n] z^{-n-1}$, $a[n] = a t^n \in \hat{\mathfrak{g}}$.
   (a) Show that if $V$ is a highest weight representation of $\hat{\mathfrak{g}}$ then $a(z)$ defines a linear map $V \to V((z))$ (here $V((z))$ is the space of formal Laurent series with coefficients in $V$).
   (b) Let $V$ have a highest weight vector $v$ with $hv = 0$ for $h$ the Cartan subalgebra of $\mathfrak{g}$, and $Kv = kv$. Compute $\langle v, a(z_1) b(z_2) v \rangle$ (as a rational function).
   (c) Compute $\langle v, a(z_1) b(z_2) c(z_3) v \rangle$. 

18.747: Problem set 6; due Thursday, April 3