18.747: Problem set 11; due Thursday, May 1

1. Let λ be a dominant integral weight for a Kac-Moody algebra $\mathfrak{g}$, and L_λ be the irreducible module with highest weight λ. Show that L_λ is the quotient of the Verma module M_λ by the submodule generated by the vectors $f_i^{\lambda(h_i)+1} v_\lambda$ (where v_λ is the highest weight vector).

2. Show that the tensor product of integrable highest weight modules over a Kac-Moody algebra is a direct sum of irreducible modules.