Robust benchmarking in noisy environments

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Abstract—We propose a benchmarking strategy that is robust in the presence of timer error, OS jitter and other environmental fluctuations, and is insensitive to the highly nonideal statistics produced by timing measurements. We construct a model that explains how these strongly nonideal statistics can arise from environmental fluctuations, and also justifies our proposed strategy. We implement this strategy in the BenchmarkTools Julia package, where it is used in production continuous integration (CI) pipelines for developing the Julia language and its ecosystem.

I. INTRODUCTION

Authors of high performance applications rely on benchmark suites to detect and avoid program regressions. However, many developers often run benchmarks and interpret their results in an ad hoc manner with little statistical rigor. This ad hoc interpretation wastes development time and can lead to misguided decisions that worsen performance.

In this paper, we consider the problem of designing a language- and platform-agnostic benchmarking methodology that is suitable for continuous integration (CI) pipelines and manual user workflows. Our methodology especially focuses on the accommodation of benchmarks whose expected execution times are short enough that timing measurements are vulnerable to error due to insufficient system timer accuracy (generally on the order of microseconds or shorter).

A. Accounting for performance variations

Modern hardware and operating systems introduce many confounding factors that complicate a developer’s ability to reason about variations in user space application performance. Consecutive timing measurements can fluctuate, possibly in a correlated manner, in ways which depend on a myriad of factors such as environment temperature, workload, power availability, and network traffic, and operating system (OS) configuration.

There is a large body of research on system quiescence aiming to identify and control for individual sources of variation in program run time measurements, each of which must be ameliorated in its own way. Many factors stem from OS behavior, including CPU frequency scaling, address space layout randomization (ASLR), virtual memory management, differences between CPU privilege levels, context switches due to interrupt handling, activity from system daemons and cluster managers, and suboptimal process- and thread-level scheduling. Even seemingly irrelevant configuration parameters like the size of the OS environment can confound experimental reproducibility by altering the alignment of data in memory. Other sources of variation come from specific language features or implementation details. For example, linkers for many languages are free to choose the binary layout of the library or executable arbitrarily, resulting in non-deterministic memory layouts. This problem is exacerbated in languages like C++, whose compilers introduce arbitrary name mangling of symbols. Overzealous compiler optimizations can also adversely affect the accuracy of hardware counters, or in extreme cases eliminate key parts of the benchmark as dead code. Yet another example is garbage collector performance, which is influenced from system parameters such as heap size.

1A summary of these factors can be found in the BenchmarkTools documentation in the docs/linuxtips.md file.
B. Statistics of timing measurements are not i.i.d.

The existence of many sources of performance variation result in timing measurements that are not necessarily independent and identically distributed (i.i.d.). As a result, many textbook statistical approaches fail due to reliance on the central limit theorem, which does not generally hold in the non-i.i.d. regime. In particular, empirical program timing distributions are also often heavy-tailed, and hence contain many outliers that distort measures of central tendency like the mean, and are not captured in others like the median.

The violation of the central limit theorem can be seen empirically in many Julia benchmarks. For example, Figure 1 shows that none of the four illustrative benchmarks considered in this paper exhibit normality in the sample mean. Instead, we see that the mean demonstrates skewed density and outliers in the first benchmark, bimodality in the second and fourth benchmarks, and upward drift in the third and fourth benchmarks.

Many other authors have also noted the lack of textbook statistical behavior in timing measurements [14]–[17]. Authors have also noted the poor statistiscal power of standard techniques such as F-tests or Student t-tests for benchmark timings [10], [13], [17], [19]. Parametric outlier detection techniques, such as the 3-sigma rule used in benchmarking software like AndroBench [20], can also fail when applied to non-i.i.d. timing measurements.

There is a lack of consensus over how non-ideal timing measurements should be treated. Some authors propose automated outlier removal and analyzing the remaining bulk distribution [20]; however, these methods run the risk of fundamentally distorting the true empirical distribution for the sake of normal analysis. Other authors have proposed purposely introducing randomness in the form of custom OS kernels [21], [22], custom compilers providing reproducible [11] or consistently randomized [23] binary layouts, or low-variability garbage collectors [24]. Unfortunately, these methods are specific to a single programming language, implementation, and/or platform. Furthermore, these methods often require administrative privileges and drastic modifications to the benchmarking environment, which are impractical to demand from ordinary users.

C. Existing benchmarking methodologies

While it is impossible to eliminate performance variation entirely [17], [25], benchmarking methodologies that attempt to account for both measurement error and external sources of variation do exist. For example, the Haskell microbenchmarking package criterion [26] attempts to thwart error due to timer inaccuracy by growing the number of benchmark executions per timing measurement as more timing measurements are obtained. After all measurements are taken, a summary estimate of the benchmark run time is obtained by examining the derivative of the ordinary least squares regression line at the point of a single evaluation point. There are three disadvantages to this approach. First, the least squares fit is sensitive to outliers [27] (though criterion does warn the user if outliers are detected). Second, measurements made earlier in this experiment are highly vulnerable to timer error, since few benchmark repetitions are used. These early measurements can skew the regression, and hence also skew the final run time estimate. Third, measurements made later in the experiment can repeat the benchmark more times than are necessary to overcome timer error, constituting an inefficient use of experiment time.

Another approach focuses on eliminating “warm-up”, assuming that first few runs of a benchmark are dominated by transient background events that eventually vanish and the timing measurements eventually become i.i.d. [19]. Their approach is largely platform-agnostic, recognizes the pitfalls of inter-measurement correlations, and acknowledges that merely increasing the number of benchmark repetitions is not always a sufficient strategy to yield i.i.d. samples. However, the assumption (based on common folklore) that benchmarks exhibit warm-up is often false, as is clear from Fig. 1 and elsewhere [17]: even Ref. [19] itself resorts to ad hoc judgment to work around the lack of a distinct warm-up phase. There is also no reason to believe that even if warm-up were observed, that the post-warm-up timings will be i.i.d. Furthermore, the authors do not report if their statistical tools generate correct confidence intervals. The moment-corrected formulae described are accurate only for near-normal distributions, which is unlikely to hold for the kinds of distributions we observed in real world statistics. Additionally, the methodology requires a manual calibration experiment to be run for each benchmark, compiler, and platform combination. As a result, this method is difficult to automate on the scale of Julia’s standard library benchmark suite, which contains over 1300 benchmarks, and is frequently expanded to improve performance test coverage.

Below, we describe our methodology to benchmarking for detecting performance regressions, and how it is justified from a microscopic model for variations in timing measurements. To the best of our knowledge, our work is the first benchmarking methodology that can be fully automated, is robust in its assumption of non-i.i.d. timing measurement statistics, and makes efficient use of a limited time budget.

II. TERMS AND DEFINITIONS

- $P_0$, $P$, $Q_0$, and $Q$ denote benchmarkable programs, each defined by a tape (sequence) of instructions.
- $I_{j}^{p}$ is the $j^{th}$ instruction in the tape defining program $P$.
- Instructions are indexed in bracketed superscripts, $[.]$.
- $D_{i}$ is the delay instruction associated with $I_{i}^{p}$.
- Delay instructions are defined in Sec. III.
- $T_{i}$ is a timing measurement, namely the amount of time taken to perform $n_{i}$ executions of a benchmarkable program. This quantity is directly measurable in an experiment.
- $t$ is a theoretical execution time. $t_{P_0}$ is the minimum time required to perform a single execution of $P_0$ on a given computer.
• **Estimated quantities** are denoted with a hat, ˆ. For example, ˆtP0 is an estimate of the theoretical execution time tP0.

• A benchmark **experiment** is a recipe for obtaining multiple timing measurements for a benchmarkable program. Experiments can be executed to obtain **trials**. The i-th trial of an experiment is a collection of timing measurements T(1)i,..., T(j)i, ..., T(k)i. Trial indices are always written using embraced superscripts, i(i).

• τ denotes time quantities that are external to the benchmarkable program:
  - τbudget is the **time budget** for an experiment.
  - τacc is the **accuracy** of the system timer, i.e. an upper bound on the maximal error in using the system timer to time an experiment.
  - τprec is the **precision** of the system timer, namely the smallest nonzero time interval measurable by the timer.

• xP(i)[j]τ(i) is the **time delay** due to the j-th **delay factor** for delay instruction D[j]. Specifically, τ(i) is the factor’s **time scale** and xP(i)[j] is the factor’s **trigger coefficient**, as introduced Sec. III. Delay factors are indexed with parenthesized superscripts, (i).

• ε is the measurement error due to timer inaccuracy.

• Em = Tm − tP0 is the total contribution of all delay factors found in measurement m, plus the measurement error ε.

• XP(i) is the **total trigger count** of the i-th delay factor during the execution of program P.

• ν is an **oracle function** that, when evaluated at an execution time t, estimates an appropriate n necessary to overcome measurement error due to τacc and τprec. The oracle function is described in detail in Sec. IV-C.

III. A MODEL FOR BENCHMARK TIMING DISTRIBUTIONS

We now present a statistical description of benchmark programs’ behavior when they are run in serial. Our model deliberately avoids the problematic assumption that timing measurements are i.i.d. We will use this model later to justify the design of a new automated experimental procedure.

A. **User benchmarks run with uncontrollable delays**

Let P0 be a deterministic benchmark program which consists of an instruction tape consisting of k instructions:

\[ P_0 = [I^{[1]}, I^{[2]}, \ldots, I^{[k]}]. \] (1)

Let τ[i] be the run time of instruction I[i]. Then, the total run time of P0 can be written \( t_{P_0} = \sum_{i=1}^{N} \tau[i] \).

While a computer may be directed to execute P0, it may not necessarily run the program’s instructions as they are originally provided, since the environment in which P0 runs is vulnerable to the factors described earlier in Sec. I-A. Crucially, these factors only delay the completion of the original instructions, rather than speed them up. Therefore, we call them **delay factors**; they can be modeled as extra instructions which, when interleaved with the original instructions, do not change the semantics of P0, but still add to the program’s total run time. Thus, we can define a new program P which consists of P0’s original instructions interleaved with additional **delay instructions** D[i]:

\[ P = [I^{[1]}, D^{[1]}, I^{[2]}, D^{[2]}, \ldots, I^{[k]}, D^{[k]}]. \] (2)

The run time of P can then be written

\[ t_P = t_{P_0} + \sum_i \tau_D[i], \] (3)

where \( \tau_D[i] \) is the execution time of D[i]. Since \( \tau_D[i] \geq 0 \), it follows that \( t_P \geq t_{P_0} \).

The run time of each delay instruction, \( \tau_D[i] \), can be further decomposed into the runtime contributions of individual delay factors. Let us imagine that each delay factor j can either contribute or not contribute to \( D[i] \). Assuming that each delay factor triggers inside \( D[i] \) with constant probability \( p[i][j] \) of taking a fixed time \( \tau(j) \), we can then write:

\[ \tau_D[i] = \sum_j x_P[i][j] \tau(j), \] (4)

where \( x_P[i][j] \) is a Bernoulli random variable with success probability \( p[i][j] \). We denote the total number of times the i-th delay factor was triggered during the execution of P as the **trigger count** \( X_P[i] = \sum_j x_P[i][j] \). Since the trigger count is a sum of independent Bernoulli random variables with nonidentical success probabilities, \( X_P[i] \) is itself a random variable that follows a Poisson binomial distribution parameterized by the success probabilities \( p[i][1], \ldots, p[i][k] \). Our final expression for \( t_P \) in terms of these quantities is then:

\[ t_P = t_{P_0} + \sum_{i=1}^{k} \tau_D[i] \]

\[ = t_{P_0} + \sum_{i=1}^{k} \sum_j x_P[i][j] \tau(j) \]

\[ = t_{P_0} + \sum_j X_P[j] \tau(j). \] (5)

In summary, our model treats \( t_P \) as a random variable whose distribution depends on the trigger probabilities \( p[i][j] \), which are determined by the combined behavior of the delay factors and the initial benchmark program P0.

B. **Repeated benchmark execution is often necessary but not always sufficient**

As mentioned in Sec. I-C, experiments which measure program performance usually incorporate multiple benchmark

While there are a very few external factors which might speed up program execution, such as frequency scaling, they can be easily accounted for by ensuring that power consumption profiles are always set for maximal performance. We therefore assume that these factors have been accounted for.
executions to obtain more accurate measurements. We now apply our model to show that multiple executions are necessary to eliminate error due to timer inaccuracy, but are insufficient to obviate delay factors.

Represent \( n \) executions of the program \( P_0 \) comprised of \( k \) instructions as a single execution of a program \( Q_0 \), which is the result of concatenating \( n \) copies of \( P_0 \):

\[
Q_0 = [P_0, P_0, \ldots, P_0] = [I_0^{[1]}, I_0^{[2]}, \ldots, I_0^{[k]}, I_0^{[1]}, I_0^{[2]}, \ldots, I_0^{[k]}] = [I_Q^{[1]}, I_Q^{[2]}, \ldots, I_Q^{[nk]}],
\]

with \( I_Q^{[i]} = I_Q^{[i+ck]} \) for \( c \in \{0, \ldots, n-1\} \). The subscripts on \( I \) denote the program which contains that instruction, with the 0 subscript dropped for brevity.

Now interleave delay instructions as before to obtain the benchmark many times. Whether or not increasing \( n \) eliminates bias due to external variations simply by executing \( n \) repetitions of \( P \) is insufficient to eliminate error due to timer inaccuracy, but are insufficient to obtain our model to show that multiple executions are necessary to obtain more accurate measurements. We now apply our model to show that multiple executions are necessary to eliminate error due to timer inaccuracy, but are insufficient to obviate delay factors.

IV. AN AUTOMATED PROCEDURE FOR CONFIGURING PERFORMANCE EXPERIMENTS

In this section, we present an experimental procedure for automatically selecting useful values of \( n \) for a given benchmark program, which can be justified from our model of serial benchmark execution above. Our procedure estimates a value for \( n \) which primarily minimizes error in timing measurements and secondarily maximizes the number of measurements obtainable within a given time budget.

A. An algorithm for estimating the optimal \( n \) value

Given \( P_0 \) and a total time budget \( \tau_{\text{budget}} \), we use the automatable procedure in Alg. I for guessing the minimum value of \( n \) required to amortize measurement error due to timer inaccuracy. The algorithm makes use of an oracle function \( \nu \), which is discussed in greater detail below in Sec. IV-C.

Algorithm 1: Estimating \( n \), the optimal number of benchmark repetitions required to minimize timer error and maximize the number of data points obtainable within a time budget.

**Input:** \( P_0, \tau_{\text{acc}}, \tau_{\text{prec}} \), an oracle function \( \nu : t \to n \)

**Output:** \( n \)

1. Let \( j = \tau_{\text{acc}}/\tau_{\text{prec}} \).
2. For \( i \in \{1, \ldots, j\} \), measure the amount of time it takes to perform \( i \) executions of \( P_0 \), resulting in a collection of timing measurements \( T_1, \ldots, T_j \).
3. Estimate \( t_{P_0} \) as \( t_{P_0} = \min(T_1, \ldots, T_j) \).
4. Evaluate \( \nu(t_{P_0}) \) to obtain \( n \). Details of \( \nu \) are given in Sec. IV-C.

The upper bound \( j \) in Alg. I is the ratio of timer accuracy to timer precision. If each timing measurement consists of more than \( j \) repetitions, then the contribution of timer inaccuracy to the total error is less than \( \tau_{\text{acc}}/j = \tau_{\text{prec}} \), and so is too small to measure. Thus, there is no reason to pick \( n > j \). In practice, \( \tau_{\text{acc}} \) need only be an overestimate for the timer accuracy, which would raise the \( n \) determined, but is still an acceptable result.

Alg. I need only be applied once per benchmark, since the estimated \( n \) can be cached for use in subsequent experiments on the same machine. Thus, we consider this algorithm an automated preprocessing step that does not count against our time budget \( \tau_{\text{budget}} \). In this regard, our approach differs significantly from other approaches like criterion, which re-determines \( n \) every time a benchmark is run.

B. Justifying the minimum estimator

We will now justify Alg. I’s use of the minimum to estimate \( t_{P_0} \), as opposed to the more common median or mean. Consider the total error term for a given timing measurement \( E_{\text{tm}} = \left( \sum_i x_Q^{[j]} \cdot \tau(i) + \epsilon \right) / n_m \), such that \( T_i / n_m = t_{P_0} + E_i \). The minimum estimator applied to our timing measurements can then be written as:

\[
\gamma_{\text{tm}} = \min\left\{ T_i / n_m \right\} = \min\left\{ t_{P_0} + E_i \right\} = t_{P_0} + \min\left\{ E_i \right\}.
\]
executions per measurement

\[ \hat{t}_{P_0} = \min\left( \frac{T_1}{T}, \ldots, \frac{T_j}{T} \right) \]
\[ = t_{P_0} + \min(E_1, \ldots, E_j). \] (10)

Thus, \( \hat{t}_{P_0} \) is the estimate of \( t_{P_0} \) which minimizes the error terms appearing in our sample.

In the limit where the delay factor time scales are greater than \( \tau_{\text{acc}} \), the total error terms will always be positive, such that choosing the smallest timing measurement will choose the sample with the smallest magnitude of error. If the delay factor time scales are less than \( \tau_{\text{acc}} \), choosing the smallest timing measurement might choose a sample which underestimates \( t_{P_0} \) due to negative timer error. In this case, Alg. 1 will simply pick a larger \( n \) than is strictly necessary, which is still acceptable.

Figs. 3 and 4 provide further justification for the minimum over other common estimators like the median, mean, or trimmed mean. Recall from Section III that the error terms \( E_i \) are sampled from a sum of scaled random variables following nonidentical Poisson binomial distributions. As such, these terms can and do exhibit multimodal behavior. While estimators like the median and trimmed mean are known to be robust to outliers \( [27] \), Fig. 3 demonstrates that they still capture bimodality of the distributions plotted in Fig. 4. Thus, these estimators are undesirable for choosing \( n \), since the result could vary drastically between different executions of Alg. 1 depending on which of the estimator’s modes was captured in the sample, and hence affect reproducibility. In contrast, the distribution of the minimum across all experimental trials is unimodal in all cases we have observed. Thus for our purposes, the minimum is a unimodal, robust estimator for the location parameter of a given benchmark’s timing distribution.

C. The oracle function

Our heuristic takes as input an oracle function \( \nu(t) \) that maps expected run times to an optimal number of executions

\[
\begin{align*}
\text{time (ns)} & \quad 0 & 20 & 40 & 60 & 80 & 100 \\
\text{Trial Index } i & \quad 170 & 180 & 190 & 200 & 210 & 220
\end{align*}
\]

\[
\begin{align*}
\text{time (ns)} & \quad 0.00 & 0.05 & 0.10 & 0.15 & 0.20 \\
\text{probability density (ns}^{-1}) & \quad 0.00 & 0.05 & 0.10 & 0.15 & 0.20
\end{align*}
\]

\[
\text{time (ns)} & \quad 175 & 180 & 185 & 190 & 195 & 200
\]

\[
\text{probability density (ns}^{-1}) & \quad 0.00 & 0.05 & 0.10 & 0.15 & 0.20
\]

\[
\text{time (ns)} & \quad 175 & 180 & 185 & 190 & 195 & 200
\]

\[
\text{kernel density estimates (KDEs) of the probability density functions (pdfs) across 100 trials of the sumindex benchmark. Each curve is a KDE formed from a trial of 10,000 consecutively gathered timing measurements. Note that the data form two distinct clusters. A cursory investigation did not reveal any inter-trial correlations that revealed a predictable preference for which cluster would be observed.}
\]
per measurement. While Alg. 1 does not directly describe \( \nu(t) \), appropriate choices for this function should have the following properties:

- \( \nu(t) \) has a discrete range \( \{1, \ldots, j\} \).
- \( \nu(t) \) is monotonically decreasing, so that the longer the run time, the fewer repetitions per measurement.
- \( \frac{\partial \nu}{\partial t} |_{t = \tau_{\text{prec}}} \approx 0 \), so that there is only weak dependence on the timer precision parameter, which may not be accurately known.
- \( \frac{\partial \nu}{\partial t} |_{t = \tau_{\text{acc}}} \approx 0 \), so that there is only weak dependence on the timer accuracy parameter, which may not be accurately known.
- \( \nu(\tau_{\text{prec}}) \approx j \), so that benchmarks that take a short time to run are not repeated more times than necessary to mitigate timer inaccuracy.
- \( \nu(t \geq \tau_{\text{acc}}) \approx 1 \), so that benchmarks that take a long time to run need not be repeated.

There are many functions that satisfy these criteria. One useful example takes the form of the generalized logistic function:

\[
Y(t) = \frac{1 + \frac{j - 1}{1 + e^{a(t-b\tau_{\text{acc}})}}}{1} \tag{11}
\]

where reasonable values of \( a \) and \( b \) are approximately \( 0.005 < a\tau_{\text{prec}} < 0.02 \) and \( 0.4 < b < 0.6 \).

In practice, we have found that better results can be achieved by first approximating \( Y(t) \) with a lookup table, then modifying the lookup table based on empirical observations. This was accomplished by examining many benchmarks with a variety of known run times at different time scales, seeking for each run time the smallest \( n \) value at which the minimum estimate appears to converge to a lower bound (e.g. around \( n = 250 \) for the benchmark in Fig. 2). Fig. 5 plots both (11) and an empirically obtained lookup table as potential oracle functions.

V. IMPLEMENTATION IN JULIA

The experimental methodology in this paper is implemented in the BenchmarkTools Julia package. In addition to the BaseBenchmarks and Nanosoldier packages, the BenchmarkTools package implements the on-demand CI benchmarking service used by core Julia developers to compare the performance of proposed language changes with respect to over 1300 benchmarks. Since this CI benchmarking service began in early 2016, it has caught and prevented the introduction of dozens of serious performance regressions into Julia’s standard library (defining a serious regression as a 30% or greater increase in a benchmark’s minimum execution time).

The benchmarks referenced in this paper are Julia benchmarks written and executed using BenchmarkTools. A brief description of each benchmark is offered below:

- The `sumindex(a, ind)` benchmark sums over all \( a[i] \) for all \( i \) in `inds`. This test stresses memory layout via element retrieval.
- The `pushall!(a, b)` benchmark pushes elements from \( b \) into a one by one, additionally generating a random number at each iteration (the random number does not affect the output). This test stresses both random number generation and periodic reallocation that occurs as part of Julia’s dynamic array resizing algorithm.
- The `branchsum(n)` benchmark loops from 1 to \( n \). If the loop variable is even, a counter is decremented. Otherwise, an inner loop is triggered which runs from 1 to \( n \), in which another parity test is performed on the inner loop variable to determine whether to increment or decrement the counter. This test stresses periodically costly branching within loop iterations.
- The `manyallocs(n)` allocates an array of \( n \) elements, where each element is itself an array. The inner array length is determined by a random number from 1 to \( n \), which is regenerated when each new array is constructed. However, the random number generator is reseeded before each generation so that the program is deterministic. This test stresses random number generation and the frequent allocation of arrays of differing length.

The mock benchmark suite referenced in this paper is hosted on GitHub at [https://github.com/JuliaCI/BenchmarkTools.jl](https://github.com/JuliaCI/BenchmarkTools.jl) and [https://github.com/JuliaCI/Backbenchmarks.jl](https://github.com/JuliaCI/Backbenchmarks.jl) and [https://github.com/JuliaCI/Nanosoldier.jl](https://github.com/JuliaCI/Nanosoldier.jl)

VI. CONCLUSION

The complexities of modern hardware and software environments produce variations in benchmark timings, with highly nonideal statistics that complicate the detection of performance regressions. Timing measurements taken from real Julia benchmarks confirm the observations of many other authors showing highly nonideal, even multimodal behavior, exhibited by even the simplest benchmark codes.

Virtually all timing variations are delays caused by flushing cache lines, task switching to background OS processes, or...
similar events. The simple observation that variations never reduce the run time led us to consider a straightforward analysis based on a simple model for delays in a serial instruction pipeline. Our results suggest that the minimum, rather than the mean or median, is the most suitable run time estimator for determining the optimal number of executions per measurement for a given benchmark. Our model also revealed some behaviors that challenge conventional wisdom: simply running a benchmark for longer, or repeating its execution many times, can render the effects of external variation negligible, even as the error due to timer inaccuracy is amortized.

Alg. 1 presents an automatable heuristic for selecting the minimum number of executions of a benchmark per measurement required to defeat timer error. This strategy has been implemented in the BenchmarkTools Julia package, which is employed daily and on demand as part Julia’s continuous integration (CI) pipeline to evaluate the performance effects of proposed changes to Julia’s standard library in a fully automatic fashion. BenchmarkTools can also be used to test the performance of user-authored Julia packages.

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