18.700 Problem Set 5

Due in class Monday November 20; late work will not be accepted. Your work on graded problem sets should be entirely your own.

1. (8 points) Let V be the real vector space consisting of all functions on the real line of the form $p(x)e^{-x^2}$, with p a polynomial of degree at most 5. If $f \in V$, define a new function Tf by

$$Tf = \frac{d^2 f}{dx^2} - 4x^2 f.$$

You more or less showed on the last problem set that T is a linear operator on V. Find all the eigenvalues of T, and decide whether T is diagonalizable.

2. (7 points) Suppose A is a 3×3 real matrix. Prove that A has at least one (real) eigenvector.

3. (8 points) Suppose the ground field is the complex numbers, θ is a real number, and

$$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

Find all the eigenvalues and eigenvectors of R, and show how to change bases to diagonalize R.

4. (7 points) Suppose the ground field is the real numbers, θ is a real number, and

$$R = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}.$$

Find all the eigenvalues and eigenvectors of R, and show how to change bases to diagonalize R.