18.700 Problem Set 1

Due in class Friday September 22; late work will not be accepted. Your work on graded problem sets should be entirely your own. In every problem the field of scalars is the real numbers.

1. (10 points) In this problem the unknowns are called a_0, a_1, a_2, and a_3.

a) Solve the system of equations

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 9 \\ 25 \end{pmatrix}$$

b) Find a cubic polynomial in t whose values at $t = 0, 1, 2, 3$ are $1, 3, 9, 25$.

2. (10 points) Suppose A is an $m \times n$ matrix, and that $m < n$. For each statement below, tell whether it is true or false; give a counterexample if it is false; and explain why if it is true.

a) The equation $AX = 0$ has infinitely many solutions.

b) There is a constant C in $\mathbb{R}^m$ so that the equation $AX = C$ has no solutions.

c) If the equation $AX = C$ has one solution, then it has infinitely many.

3. (10 points) I have in my pocket 11 coins: a mixture of pennies, nickels, and dimes worth 66 cents.

a) How many coins of each type do I have?

b) Explain why the fact that this problem has a unique solution does not contradict Problem 2.