Background about Langlands classification:

Parameters \( \gamma \) \( \rightarrow \) STANDARD REP \( \pi(\gamma) \)

More or less \( \Rightarrow \) LANGLANDS QUOTIENT \( \pi(\gamma) \)

Chaos of CSGs

\[
\pi(\gamma) = \pi(\gamma) + \sum_{\text{other } \gamma'} m(\gamma', \gamma) \pi(\gamma')
\]

PARAMETERS have height \( \gamma \)

Size of discrete series parameter that's part of \( \gamma \)

Size of character \( \gamma \)

Compact part of torus

Parameter with height \( \gamma \)

Size of discrete series

Integer \( \geq 0 \)

MULTIPLICITY

\( \gamma, \gamma' \) SAME INFL CHAR

block of \( \pi(\gamma) \) \( \rightarrow \) all parameters \( \gamma' \)

Finitely indexed by \( B(\gamma) \)

\[
m \otimes B \times B \rightarrow N
\]

Matrix: indexed by \( B \)

\( m(\gamma', \gamma) = 0 \) unless height \( \gamma' \) \( \geq \) height \( \gamma \)

upper \( \Delta \) is on diagonal

Computation of \( m \) by KL theory
**UNITARITY?**

\[ x = (\lambda, \nu) \]

Continuous part

Discrete series part

\( HE/G \uparrow \)

\[ y_t = (\lambda, t, \nu) \quad 0 \leq t \leq 1 \]

\[ I(x_0) = \text{UNITARILY INDUCED}; \]

POS def invt form.

Study unitarity. Look at form \( \langle , \rangle_t \)

Of \( I(y_t) \): Changes signature only at REDUCIBLE \( I(y_t) \) (form NON DEG when \( I(y_t) \) irrel.) ATLAS: finds reducibility points \( t \ldots t_m \in [0, 1] \) (FINITE)

\[ \begin{bmatrix} 0 & \emptyset & 0.5 \nu \\ 0.5 \nu & F & F \end{bmatrix} \]

For which \( t \) is \( J(t) \) unitary?

Answer: changes only at \( \frac{1}{3}, \frac{1}{2} \)

Get different answers at \( 0, (0, \frac{1}{3}), \frac{1}{3}, (\frac{1}{3}, \frac{1}{2}), \frac{1}{2}, (\frac{1}{2}, 1), 1 \)

**CSG TA**
Software starts with pos form at \( I(y) \)

\[
J(y') \\
s' \text{ comp factor of } I(y', t) \\
\text{same height as orig } y \text{ (degs on } A) \\
\]

\[
\text{CHANGE at } I(t, y') \\
\text{deforms bigger height} \\
\text{forms on higher comp factors of } I(t, y') \\
\text{know init Herm form by downward induction on height} \\
\]

\[
\text{each } y' \Rightarrow y \text{ in block, find all red pts } t_1 \ldots t_m(y') \\
\text{get more parameters} \\
\text{BIGGER height} \\
\text{smaller infl char} \\
\text{ALL } y'' \text{ have smaller infl char} \\
\text{than } y \rightarrow \text{finite set of } y'' \\
\]
Y is giant finite set $\mathcal{D}$ of params (bigger height, smaller inf \ char). For each, write FORMULA for inv Herm form

$$\langle , \rangle \mathcal{D} = \left( \sum_{x''} p(x'')I(x'',0), \sum_{x''} q(x'')I(x'',0) \right)$$

form on $\mathcal{J}(x'')$

Write form as integer comb of (definite) forms on temporal standards ($x'' = 0$)

$p(x''), q(x'') \in \mathbb{Z}$

Store all these $x''$ = giant vector of parameters

Store for each $x''$ formula

$$\sum_{x''} \left[ p(x'') + sq(x'') \right] I(x'')$$

in $W = \mathbb{Z} + s\mathbb{Z}$

COMPUTATIONS use KL polys

for all blocks $B_1, B_2, B_3$ --- big collection of blocks of parameters