Background about Langlands classif.

{parameters γ} $\longrightarrow$ STANDARD REP $I(\gamma)$

more or less

chars of CSGs

(irr) LANGLANDS QUOTIENT $J(\gamma)$

$$I(\gamma) = J(\gamma) + \sum_{\text{other } \gamma'} \underbrace{m(\gamma';\gamma)}_{\text{integer} \geq 0} J(\gamma')$$

comp series

MULTIPLICITY

height$(\gamma') >$ height(γ)

γ', γ SAME INFL CHAR

PARAMETERS have height(γ)

SIZE OF discrete series parameter that's part of γ

size of character $|_{T^\Theta}$ ← compact part of torus

Give atlas param $\gamma \longrightarrow$ block_of(γ) ← all parameters γ' same infl char as γ + more reqts

FINITE $\to B(\gamma)$

$m : B \times B \to \mathbb{N}$

mult fn

MATRIX: indexed by B

$m(\gamma';\gamma) = 0$ unless height$(\gamma') \geq$ height of γ

upper Δ

1's on diagonal. COMPUTE m by KL theory

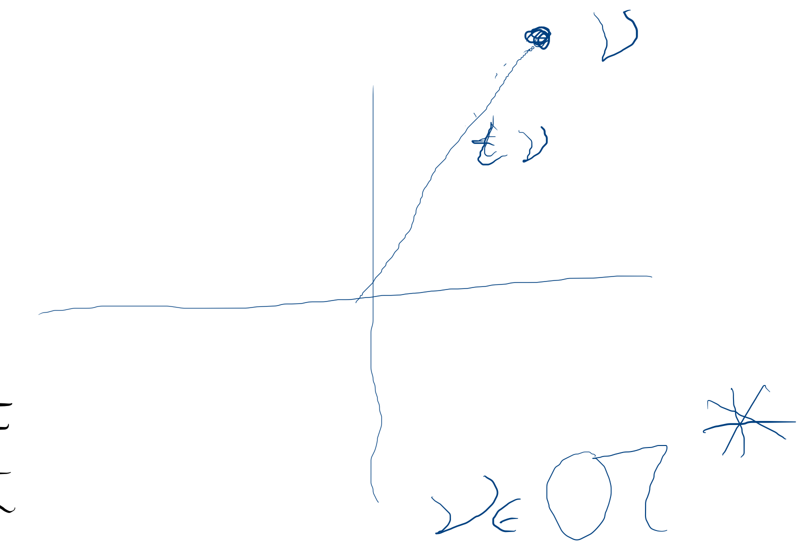
UNITARITY?

$\gamma = (\lambda, \nu)$

λ: discrete series part (HEIGHT)

ν: continuous part

$\boxed{\gamma_t = (\lambda, t\nu) \quad 0 \le t \le 1}$



CSG TA

UNITARILY INDUCED:

$I(\gamma_0)$ = pos def invt form.

study unitarity: ~~see~~ look at form $\langle\, ,\rangle_t$ of $I(\gamma_t)$: Changes signature only at REDUCIBLE $I(\gamma_t)$ (form NON DEG when $I(\gamma_t)$ irr.) ATLAS: finds reducibility points $t_1 \dots t_m \in [0,1]$ (FINITE)

$[0 \quad \cdot \quad \tfrac{1}{3}\nu \quad ? \quad \tfrac{1}{2}\nu \quad \cdots \quad \nu]$

T T T F F T

For which t is $J(t\nu)$ unitary?

Answer changes only at $\tfrac{1}{3}\nu$, $\tfrac{1}{2}\nu$

Get different answers at 0, $(0, \tfrac{1}{3})$, $\tfrac{1}{3}$, $(\tfrac{1}{3}, \tfrac{1}{2})$, $\tfrac{1}{2}$, $(\tfrac{1}{2}, 1)$, 1

Software starts with pos form at $I(0,\nu)$ $\xrightarrow{\text{deforms}}$ CHANGE at $I(t,\nu)$

forms on higher comp factors of $I(t,\nu)$ height

bigger height

$J(\gamma')$, γ' compfactor of $I(\lambda, t, \nu)$

KNOW invt Herm form by downward induction on height

same height as orig γ (deps on λ)

γ $B(\gamma)$ Each $\gamma' > \gamma$ in block, find all red pts $t_1 \ldots t_{m(\gamma')}$

Get more parameters γ'' $\leftarrow$ comp factors of $I(\gamma', t, \nu')$

BIGGER height smaller infl char

ALL γ'' have smaller infl char bigger height than γ $\longrightarrow$ FINITE SET of γ''

$\gamma \to$ giant finite set $\mathcal{Q}$ of params (bigger height, smaller infl char. For each, write FORMULA for invt Herm form

$$\underbrace{\langle\, , \,\rangle_{\gamma''}}_{\text{form on } J(\gamma'')} = \left(\sum_{\cancel{\lambda}'''} p(\cancel{\lambda}''') I(\cancel{\lambda}''', 0), \sum_{\cancel{\lambda}'''} q(\cancel{\lambda}''') I(\lambda''', 0) \right)$$

Write form as integer comb of (definite) forms on tempered standards ($\nu''' = 0$) $p(\lambda'''), q(\lambda''') \in \mathbb{Z}$

Store all these $\gamma''' \leftarrow$ giant vector of parameters

store for each γ''' formula $\sum_{\lambda'''} \underbrace{[p(\lambda''') + s\,q(\lambda''')]}_{\text{in } W = \mathbb{Z}_{pos} + s\mathbb{Z}_{neg}} I(\lambda''')$

COMPUTATIONS use KL polys for all blocks $\mathcal{B}_0$, $\mathcal{B}_1$, $\mathcal{B}_2$ ... big collection of blocks of parameters