Background about Langlands classify.

Parameters $\gamma \rightarrow$ STANDARD REP $\pi(\gamma)$

\[ \text{more or less} \quad \frac{\text{LANGLANDS QUOTIENT } J(\gamma)}{\text{chars of CSGs}} \]

\[ I(\gamma) = J(\gamma) + \sum_{\text{other } \gamma'} m(\gamma, \gamma') J(\gamma') \]

PARAMETERS have height $\gamma$

SIZE of discrete series parameter that is part of $\gamma$

size of character $\gamma$

compact part of torus

same infl char

$\gamma, \gamma'$ same infl char

$\gamma, \gamma'$ same infl char as $\gamma + \text{more reps}$

Given atlas param $\gamma \rightarrow$ block of $(\gamma) \in$ all parameters $\gamma$, same infl char as $\gamma$ + more reps

$B(\gamma)$ finite

$B \times B \rightarrow M$

MATRICES: indexed by $B$

$m(\gamma, \gamma') = 0$ unless $\text{height}(\gamma') \geq \text{height of } \gamma$

$m \in \mathbb{N}$

upper $\Delta$ on diagonal

compute $m$ by KL theory
UNITARITY?

\[
Y = (\lambda, \nu) \\
\text{continuous part}
\]

\[
\text{discrete part (HEIGHT)}
\]

\[
I(Y_0) = \text{UNITARILY INDUCED:}
\]

POS def invt form.

study unitarity, look at form \( \langle \cdot, \cdot \rangle_t \)

of \( I(Y_t) \): Changes signature only at REDUCIBLE \( I(Y_t) \) (form NON DEG when \( I(Y_t) \) irr.) ATLAS: finds reducibility

points \( t_1, \ldots, t_m \in [0, 1] \) (FINITE)

\[
\begin{array}{c|c|c|c|c}
0 & \frac{1}{2} & 1 \\
T & T & F & F & T
\end{array}
\]

For which \( t \) is \( J(t) \) unitary?

Answer changes only at \( \frac{1}{2} \)

Get different answers at \( 0, (0, \frac{1}{2}), \frac{1}{2}, (\frac{1}{4}, \frac{1}{2}), \frac{1}{2}, (\frac{1}{2}, 1) \)
Software starts with pos form at $I(0,y)$. Deform at $I(t_1,y)$.

$J(y')$, $y'$ comp factor of $I(x,t_1)$. Bigger height same height as orig $x$ (deps on $y$).

CHANGE $I(t_1,y)$. Comps factors of $I(t_1,y)$ forms on higher height. KNOW init Herm form by downward induction on height.

$\beta(y)$. Each $y' \supseteq y$ in block, find all red pts $t_1 \ldots t_m(y')$.

Get more parameters. $y''$ comp factors of $I(y', t_j y'')$.

$y''$ BIGGER height, Smaller infl char. ALL $y''$ have Smaller infl char bigger height than $y$. FINITE SET of $y''$. 
Y = giant finite set $\mathcal{D}$ of params (bigger height)
smaller inf \ char. For each, write \textit{FORMULA} for
\textit{inv}t Herm form

$$\langle \ , \ \rangle_{y''} = \left( \sum_{x''} p(x'') I(x'', 0) \right) \sum_{x''} q(x'') I(x'', 0)$$

form on $J(x'')$
Write form as integer comb of (definite) forms on tempered
standards ($x'' = 0$)

$p(\lambda''), q(\lambda'') \in \mathbb{Z}$

Store all these $y''$ in giant vector of parameters
Store for each $x''$ formula

$$\sum_{\lambda''} \left[ p(\lambda'') + sq(\lambda'') \right] I(\lambda'')$$

in $W = \mathbb{Z} + s\mathbb{Z}$

\textit{Computations} use KL polys
for all blocks $B_1, B_2, B_3 \ldots$ big collection
of blocks of parameters
This is WHERE APRIL 7 starts!

Start with parameter $p$, infl char -> big collection of smaller infl char

$$\text{formulas } \eta(g, \nu, \ell, \varphi) = \text{sum with } \text{Mell-coeffs of tempered reps}$$

Project of Jeff Adams, Steve Miller, Marc van Leeuwen, Annette Paul

1) Compute all Arthur reps for all exceptional non-cplx $G$

DONE! weak packets ($E_8$'s)

2) Prove all these reps are unitary

$E_8$ : few hundred reps -> all but $\approx 20$? known unitary.

Reason only $E_8$ is unitary (parameter) only for $E_8$'s

is_unitary ($E_7$'s trivial)

has not finished ever. Using little memory: has been running in 2G of mem for
$\approx 2$ weeks, not close (???)

MVL has software version

FASTER/normal code: uses $\approx 200$ G

Has run out of MEM on 300G computers
CONCLUDE: need to make software FASTER or have some math ideas to prove last few reps are unitary.

NEXT WEEK

Jeff: making software list
Arthur reps