Background about Langlands classif.

{parameters γ} $\longrightarrow$ STANDARD REP $I(\gamma)$

$\downarrow$ (irr) LANGLANDS QUOTIENT $J(\gamma)$

$\updownarrow$ more or less

chars of CSGs

$$\underbrace{I(\gamma) = J(\gamma)}_{\text{comp series}} + \sum_{\text{other } \gamma'} \underbrace{m(\gamma',\gamma)}_{\substack{\text{integer} \geq 0 \\ \text{MULTIPLICITY}}} J(\gamma')$$

height(γ') > height(γ)

γ', γ SAME INFL CHAR

PARAMETERS have height(γ)

SIZE OF discrete series parameter that's part of γ

$\updownarrow$

size of character $\theta|_T$ ← compact part of torus

Give atlas param $\gamma \longrightarrow$ block_of(γ) ← all parameters γ' same infl char as γ + more reqts

FINITE $\to B(\gamma)$

$m : B \times B \to \mathbb{N}$ MATRIX: indexed by B $m(\gamma',\gamma) = 0$ unless height$(\gamma') \geq$ height of γ

mult fn

upper Δ

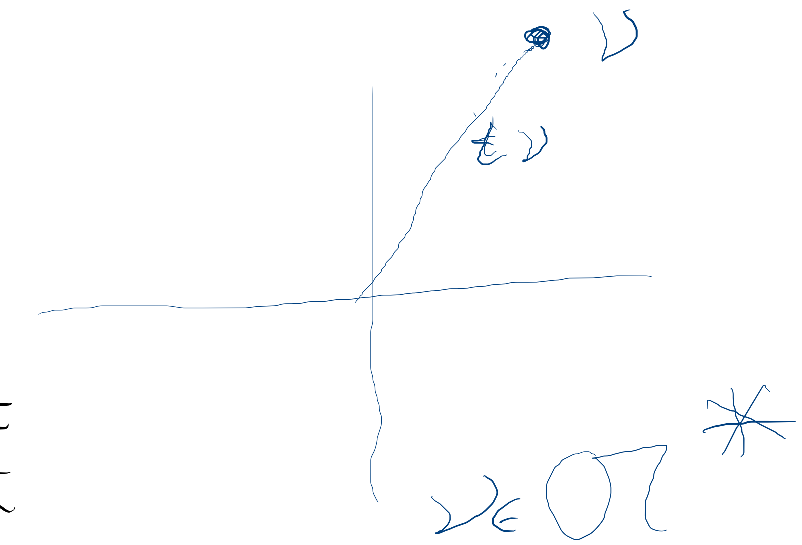
1's on diagonal. COMPUTE m by KL theory

UNITARITY?

$\gamma = (\lambda, \nu)$

discrete series part (HEIGHT) — λ; continuous part — ν

$\boxed{\gamma_t = (\lambda, t\nu) \quad 0 \le t \le 1}$



CSG TA

$I(\gamma_0)$ = UNITARILY INDUCED: pos def invt form.

study unitarity: ~~see~~ look at form $\langle\,,\,\rangle_t$ of $I(\gamma_t)$: Changes signature only at REDUCIBLE $I(\gamma_t)$ (form NON DEG when $I(\gamma_t)$ irr.) ATLAS: finds reducibility points $t_1 \dots t_m$ $\in [0, 1]$ (FINITE)

[0		? $\tfrac{1}{3}\nu$		$\tfrac{1}{2}\nu$	ν]
T	T	T	F	F	T

For which t is $J(t\nu)$ unitary? Answer changes only at $\tfrac{1}{3}\nu$, $\tfrac{1}{2}\nu$

Get different answers at 0, $(0, \tfrac{1}{3})$, $\tfrac{1}{3}$, $(\tfrac{1}{3}, \tfrac{1}{2})$, $\tfrac{1}{2}$, $(\tfrac{1}{2}, 1)$, 1

Software starts with
pos form at $I(0,\nu)$ $\longrightarrow$ deforms

CHANGE
at $I(t,\nu)$

$J(\gamma')$,
γ' compfactor of
$I(\lambda, t, \nu)$

bigger height

same height as
orig γ (deps on λ)

forms on higher height
comp factors of $I(t,\nu)$

KNOW invt Herm form by
downward induction on height

γ $B(\gamma)$ Each $\gamma' > \gamma$ in block, find all red pts $t_1 \ldots t_{m(\gamma')}$

Get more parameters γ'' $\leftarrow$ comp factors of $I(\gamma', t, \nu')$

BIGGER height
smaller infl char

ALL γ'' have smaller infl char
bigger height
than γ $\longrightarrow$ FINITE SET
of γ''

TOPIC: HOW DOES ATLAS COMPUTE SIGNATURES

$\gamma \to$ giant finite set $\mathcal{L}$ of params (bigger height) smaller infl char. For each, write FORMULA for invt Herm form

$$\underbrace{\langle\, ,\, \rangle_{\gamma''}}_{\text{form on } J(\gamma'')} = \left(\sum_{\cancel{\lambda'''}} p(\cancel{\lambda'''}) I(\cancel{\lambda'''}, 0),\ \sum_{\cancel{\lambda'''}} q(\cancel{\lambda'''}) I(\lambda''', 0) \right)$$

Write form as integer comb of (definite) forms on tempered standards ($\underline{\nu''' = 0}$) $p(\lambda'''),\ q(\lambda''') \in \mathbb{Z}$

Store all these $\gamma''' \leftarrow$ giant vector of parameters

store for each γ'' formula $\sum_{\lambda'''} \left[p(\lambda''') + s\, q(\lambda''') \right] I(\lambda''')$

in $W = \underset{\text{pos}}{\mathbb{Z}} + s \underset{\text{neg}}{\mathbb{Z}}$

COMPUTATIONS use KL polys for all blocks $\mathcal{B}, \mathcal{B}_1, \mathcal{B}_2 \ldots$ big collection of blocks of parameters

$\mathcal{B}'' = \mathcal{B}_0$

THIS IS WHERE APRIL ~~7~~ 8 starts!

Start with parameter p, infl char

$\longrightarrow$ big collection of smaller-infl-char q params

formulas Sig(irr for q) = sum with ~~W~~-coeffs

$\{m + n s\}$ — m pos, n neg

of tempered reps

Project of Jeff Adams, Steve Miller, Marc van Leeuwen, Annegret PAUL

1) Compute all Arthur reps for all exceptional non-cp'x G

DONE! wpak_packets(E8_s)

2) Prove all these reps are unitary

E_8: few hundred reps $\rightarrow$ all but $\simeq 20$? known unitary only for E_8-s

Reason only E8: is_unitary (parameter) $\rightarrow$ ANSWER eventually, with enough memory

is_unitary(E7_s.trivial) ever. MVL has software version

has not finished? running in 2GIG of mem for

using little memory: has been ~~FASTER~~ /normal code: uses $\simeq$ 200 G

$\simeq$ 2 weeks; not close (??) ~~has been running~~ HAS RUN OUT OF MEMORY on ~300G computers

CONCLUDE: need to make software FASTER or have some math ideas to prove last few reps are unitary.

NEXT WEEK

Jeff: making software list

Arthur reps