April 1: David Vogan (MIT) What’s special about special?

Around 1978, Lusztig’s investigations of the representations of finite Chevalley groups led him to single out a class of Weyl group representations he called special. The corresponding set of unipotent classes is also called special. These unipotent classes arise also in representation theory over any local field. The main result over $\mathbb{R}$ says that any representation of a real reductive group having integral infinitesimal character gives rise to a special unipotent class.

In 2018 Meinolf Geck gave a conjectural characterization of special unipotent classes by an integrality criterion, which I will recall. Geck’s conjecture was established in 2019 by Junbin Dong and Gao Yang. The point of this talk is to pose the question: can the result above (connecting integral representation theory over $\mathbb{R}$ to special unipotent elements) be directly established using Geck’s conjectural characterization of special?