September 5: "Signatures of invariant Hermitian forms on finite-dimensional representations."

Suppose G is a real reductive algebraic group, and π is an irreducible complex representation of G. It often happens that π admits a non-zero G-invariant Hermitian form $\langle \cdot, \cdot \rangle_\pi$. Schur's lemma guarantees that the form is nondegenerate and unique up to a real scalar; so Sylvester's theorem says that the only possible signatures are (p, q) and (q, p). Write $\mathrm{Sig}(\pi) = |p - q|$; the smallness of $\mathrm{Sig}(\pi)$ measures how thoroughly indefinite the form is.

The Weyl dimension formula says that $\dim(\pi)$ is a polynomial of degree equal to $(\dim G - \mathrm{rank}(G))/2$ in the highest weight. I'll prove that $\mathrm{Sig}(\pi)$ is a quasipolynomial of degree $(\dim K - \mathrm{rank}(K))/2$ in the highest weight, with K a maximal compact subgroup of G. This says (for noncompact G) that the signature is "much smaller" than the dimension, meaning that the form is very indefinite.

This is joint work with MIT undergraduate Christopher Xu and his grad student mentor Daniil Kalinov.