October 3: “On twisted Gelfand pairs through commutativity of a Hecke algebra.”

A pair $(G, H)$ of a finite group $G$ and a subgroup $H$ is called a Gelfand pair if every irreducible representation of $G$ appears at most once in the representation $\mathbb{C}[G/H]$ of complex-valued functions on $G/H$. In this case, the Gelfand property is equivalent to the commutativity of the Hecke algebra $\mathbb{C}[H\backslash G/H]$ of bi-$H$-invariant functions on $G$.

Given a reductive $p$-adic group $G$ and a closed subgroup $H$, one can generalize the Gelfand property to these settings, and a result of Gelfand and Kazhdan gives sufficient conditions for $(G, H)$ to be a Gelfand pair. Unlike the finite situation, here it is not known whether there is a characterization of the Gelfand property through commutativity of an algebra.

In this talk (based on arXiv:1807.02843), we define a Hecke algebra for the pair $(G, H)$ as above and show that if the Gelfand-Kazhdan conditions hold then it is commutative. We then explore the connection between commutativity of this algebra and the Gelfand property of $(G, H)$, and show that for spherical pairs, the cuspidal part of this algebra is commutative if and only if the pair $(G, H)$ satisfies the Gelfand property with respect to all irreducible cuspidal representations of $G$. 