April 11: David Vogan (MIT), *Nilpotent orbits and Weyl group representations continued.*

Last week I began to sketch a version of ideas of Lusztig for relating nilpotent coadjoint orbits (for a complex reductive algebraic group G) to Weyl group representations. A little more precisely, one aspect of this relationship was a surjection

$$({}^\vee L, {}^\vee\mathcal{O}) \to \mathcal{O}.$$

Here the pairs on the left consist of the centralizer ${}^\vee L$ of a semisimple element in the dual group ${}^\vee G$, and a *special* nilpotent class for ${}^\vee L$; and what is on the right is a nilpotent class for G. One construction of this surjection uses the Springer correspondences with Weyl group representations. One way to think of Lusztig's map is as a generalization of the order-reversing "transposition" map from partitions of n to partitions of n (thought of as corresponding to orbits for $GL(n)$). I will look at it more closely in some small interesting examples, including G_2.

Next, I will explain how Lusztig extends this to cover *all* Weyl group representations. A consequence is a (partly conjectural) description of the "cell representations" of Weyl groups for real reductive groups. These representations generalize the Kazhdan-Lusztig cell; their structure among other things controls the structure of Arthur's "unipotent representations" for real groups.