Groups of diffeomorphisms of a manifold $M$ have many of the properties of finite-dimensional Lie groups, but also differ in surprising ways. I review some (or all or more) of the following properties or I do something else:

- No complexification.
- Exponential mappings are defined but are not locally surjective or injective.
- Right invariant Riemannian metrics might have vanishing geodesic distance.
- Many famous PDEs arise as geodesic equations on diffeomorphism groups.
- There are topological groups of diffeomorphisms which are smooth manifolds but only right translations are smooth.
- There are diffeomorphism groups which are smooth in a certain sense (some Denjoy-ultradifferentiable class) but not better (not real analytic).