February 8 and 15: David Vogan (MIT), Software for unitary representations.

Let $G$ be a linear real reductive Lie group. By the 1980s it was understood that computing the unitary dual of $G$ amounted to answering the question

$$\text{is } \pi \text{ unitary?}$$

for a finite set of irreducible representations $\pi$ of $G$. For a fixed $\pi$, it was also understood that answering this question amounted to deciding the positivity of a finite collection of symmetric matrices with rational entries; so the answer was in principle computable. But the size of the matrices involved (for $Sp(8, C)$, the largest has size 38,625) made the calculation impractical except for a few very small groups.

Work with Jeffrey Adams, Marc van Leeuwen, and Peter Trapa, and other work with George Lusztig, provided an algorithm based on the Kazhdan-Lusztig algorithm for answering this question. This algorithm has now been implemented in the atlas software: one can enter any $G$ and $\pi$, ask whether $\pi$ is unitary, and receive an answer. In the first talk I’ll demonstrate the software, and talk about the main ideas behind it. In the second talk, I’ll discuss some of the details: why five years elapsed between the mathematical formulation of the algorithm and the completion of the software.