April 19: David Vogan (MIT), “Nilpotent orbits, $K$-types, and unitarity, suite.”

Last week I formulated some [old known] facts about the relationship (for representations of reductive groups over local fields) between germ expansions of characters and restriction of representations to maximal compact subgroups.

This week I’ll suggest some [new unproven] conjectural statements relating germ expansions to unitarity. The statements involve finite-dimensional representations (for certain algebraic subgroups) defined only near the identity. In the case of $p$-adic groups, any finite-dimensional representation is necessarily trivial near the identity, so the new statements are true but uninteresting. In the case of real groups, the statements have content. I’ll try to explain this, and to relate it to Dong’s suggestion mentioned last week:

Chaoping Dong has been using the atlas software to investigate unitary representations of complex exceptional groups. He noticed that “non-unitary representations have more $K$-types with higher multiplicities.” I will discuss how to make such a statement precise, and talk about an (entirely conjectural) way that one might try to prove it.