September 7: David Vogan (MIT), “Kazhdan-Lusztig polynomials for involutions.”

This is mostly a report of work of Lusztig.

Suppose $G$ is a complex reductive algebraic group with Weyl group $(W, S)$, and Hecke algebra $H$. Suppose that $K \subset G$ is a symmetric subgroup, the group of fixed points of an involutive automorphism $\theta$ of $G$. Write $X$ for the (finite) set of $K$-equivariant local systems on orbits of $K$ on $B$, the variety of Borel subgroups of $G$. Kazhdan and Lusztig thirty years ago introduced polynomials $P_{x,y}$ (for $x$ and $y$ in $X$) that carry subtle information about representation theory and the singularities of $K$-orbit closures. All of this is related to a Hecke algebra representation with basis $X$.

Suppose now that $\delta$ is an automorphism of $G$ preserving a pinning and commuting with $\theta$. The group of fixed points $W^\delta$ is again a Coxeter group, with generators indexed by the set $T$ of orbits of $\langle \delta \rangle$ on $S$. There is an unequal parameter Hecke algebra $\tilde{H}$ for $(W^\delta, T)$.

There is a natural action of $\delta$ on $X$; write $Z$ for the set of fixed points. Lusztig twenty-five years ago (in some cases) and recently (in general) introduced new polynomials $P_{z,z'}$ (for $z$ and $z'$ in $Z$) carrying information about representation theory and the singularities of $K$-orbit closures. All of this is related to an $\tilde{H}$ representation with basis $Z$.

A first example is $G = G_1 \times G_1$ with $\delta$ the involution interchanging the two factors and $K \simeq G_1$ the diagonal subgroup. In this case $X = W_1$, and $Z$ is the set of involutions (of order one or two) in $W_1$. The Hecke algebra $\tilde{H}$ is the Hecke algebra of $W_1$ with parameter $q^2$. The representation at $q = 1$ of $W_1$ is in type A an “involution model,” but in other types something new and somewhat different.