November 16: David Vogan (MIT), “Left cells and Harish-Chandra cells of Weyl group representations II”

(Part I actually discussed only some “complex group” preliminaries for this talk. So I have reprinted the abstract, which I’ll try actually to follow this time.)

Suppose \( G(R) \) is a real reductive algebraic group, and \( \lambda \) is a regular integral infinitesimal character. There is a finite set

\[
B(\lambda) = \{X_1, \ldots, X_n\}
\]

of irreducible representations of infinitesimal character \( \lambda \). Kazhdan-Lusztig theory provides a “\( W \)-graph” structure on \( B(\lambda) \), which among other things provides a representation of \( W \) (defined over \( \mathbb{Z} \)) with basis \( B(\lambda) \). The (directed) edges of the graph provide a preorder on \( B(\lambda) \), telling when one representation can be obtained from another as a subquotient of a tensor product with a finite-dimensional representation. The equivalence classes for the preorder are called the “Harish-Chandra cells” for \( B(\lambda) \). All the representations in a single Harish-Chandra cell have the same Gelfand-Kirillov dimension and associated variety, so it is useful to understand these cells.

Each Harish-Chandra cell inherits from \( B(\lambda) \) a \( W \)-graph structure, and therefore constitutes a basis for a representation of \( W \). The question I want to consider is this: how are Harish-Chandra cells related to the left cells defined by Kazhdan and Lusztig? This is the kind of question that the atlas software is able to study, and Birne Binegar proved in this way that for the real exceptional groups, every Harish-Chandra cell is isomorphic (as a Weyl group representation) to a Kazhdan-Lusztig left cell. For real forms of the classical Lie algebras, the same statement is true up to rank eleven; and McGovern has proved the same assertion for most of the “classical” classical groups (like \( SO(2a + 1, 2b) \)).

Nevertheless (as McGovern also showed) there are Harish-Chandra cells that are not isomorphic to Kazhdan-Lusztig left cells. I will explain his example, and formulate some new problems in this (still mysterious) direction.