March 14: David Vogan (MIT), “What the local Langlands correspondence can do for you.”

Langlands’ program (mostly conjectural) says that automorphic representations \( \pi \) on a reductive group \( G \) (defined over a number field \( k \)) should be indexed by arithmetic parameters, related to the Galois group of \( k \) and to the complex dual group \( \hat{G} \) of \( G \). It’s a theorem that \( \pi \) is a tensor product over places \( v \) of \( k \) of representations \( \pi_v \) of \( G(k_v) \). The local Langlands program (proved for \( GL_n \) and for all groups over \( \mathbb{R} \)) says that representations \( \pi_v \) should be indexed by local arithmetic parameters, related to the Galois group of \( k_v \) and to \( \hat{G} \). (Then there should be a big commutative diagram . . . )

Over a \( p \)-adic field \( k_v \), Langlands’ precise formulation of these statements was improved by Deligne and by Lusztig to a rather complete (conjectural) description of representation theory for \( G(k_v) \) in terms of (complex algebraic) geometry on \( \hat{G} \). Over \( \mathbb{R} \), Langlands precise and proven formulation was (I claim) not the right one. This talk is an advertisement for a different formulation, due (long long ago) to Adams, Barbasch, and me. Main theorem/example will be a description of “blocks” of representations of \( G(\mathbb{R}) \) (equivalence classes under the relation “has a non-trivial extension with”) in terms of (roughly speaking) real forms of \( \hat{G} \).

The number-theoretic content of this is that the definition of the real Weil group is not the right one. I’ll explain the classical definition and the replacement.