November 11: David Vogan (MIT), “Computing Hodge and weight filtrations of Harish-Chandra modules.”

Last week Wilfried Schmid explained how to define two canonical filtrations on a \((g, K)\) module: a “Hodge filtration,” which is a \(K\)-invariant good filtration and therefore defines a \(q\)-analogue of the \(K\)-multiplicities; and a “weight filtration,” which is a generalization of the Jantzen filtration of a Verma module. He also explained (joint work with Vilonen) how an understanding of these filtrations should determine the unitary dual of the corresponding reductive group.

The weight filtrations of (standard) Harish-Chandra modules were essentially computed by Beilinson and Bernstein some twenty years ago, using Kazhdan-Lusztig polynomials.

In earlier seminars I have outlined joint work with Adams, van Leeuwen, Trapa, and Yee which should determine the unitary dual. By reading the Schmid-Vilonen work backwards, our work should offer algorithms for computing Hodge filtrations of (standard) Harish-Chandra modules. I’ll explain how that should work, and progress (by Marc van Leeuwen) on writing software to implement the algorithms.