April 5: B. Kostant (MIT), “Experimental evidence for the occurrence of $E_8$ in nature and the radii of the Gossett circles.”

A recent experimental discovery involving spin structure of electrons in a cold one-dimensional magnet points to a validation of a Zamolodchikov model involving the exceptional Lie group $E_8$. The model predicts 8 particles and predicts the ratio of their masses. In more detail, the vertices of the 8-dimensional Gossett polytope identifies with the 240 roots of $E_8$. Under the famous two-dimensional (Peter McMullen) projection of the polytope, the image of the vertices are arranged in 8 concentric circles, hereafter referred to as the Gosset circles. The Gosset circles are now understood to correspond to the 8 masses in the model, and in addition it is understood that the ratio of the their radii is the same as the ratio of the corresponding conjectural masses. A ratio of the two smallest circles (read 2 smallest masses) is the golden number. Marvelously, the conjectures have been now validated experimentally, at least for the first five masses; see [Co].

The McMullen projection generalizes to any complex simple Lie algebra in particular not restricted to $A - D - E$) whose rank is greater than 1. The Gosset circles generalize as well, using orbits of the Coxeter element. Applying results in [Ko], I found a very easily defined operator $A$ on a Cartan subalgebra whose spectrum is exactly $h/2$ times the squares of the radii $r_i$ of these generalized Gosset circles. Here $h$ is the Coxeter number. The operator $A$ is written as a sum of $\ell + 1$ rank 1 operators, parameterized by the points in the extended Dynkin diagram. Involved in this expansion are the coefficients $n_i$ of the highest root. As a confirmation of the relation between Zamolodchikov’s masses and the radii of the Gossett circles, David Vogan (using Maple) computed the ratio of the $r_i$. Happily, using Vogan’s computations, Nolan Wallach showed that the ratios are exactly the ratios of Zamolodchikov’s masses. As later pointed out to me (by Wallach), a further confirmation, not using computer programs, the masses for $E_6$, $E_7$, and $E_8$ models are stated as being derived from the eigenvalues of a matrix $A$ in (1.6) in [FZ]. No mention of the radii of the generalized Gossett circles is made in either [Za] or [FZ]. Therefore, (1.6) in [FZ], together with Theorem 2.1 in our present paper, implies the ratio of masses is equal to the ratio of the radii in these cases as well.


