Let $g$ be a complex semisimple Lie algebra, and $G$ the identity component of its automorphism group. A “finite maximal torus” for $G$ is a maximal abelian subgroup $A$ of $G$ that is also finite. Such subgroups have been classified in many cases (including all the classical simple Lie algebras). Each gives rise to a grading

$$g = \sum_{\xi \in A} g_{\xi}$$

according to the characters by which $A$ acts on $g$. The roots of $A$ in $g$, $R(\mathfrak{g}, A)$ are the characters $\xi$ for which $g_{\xi} \neq 0$.

This decomposition behaves formally much like the usual root decomposition, but the finiteness of $A$ makes two great differences: the trivial eigenspace $g_1$ is zero; and each space $g_{\xi}$ is the Lie algebra of a torus (rather than a unipotent subgroup).

Just as combinatorial study of the classical root system reveals interesting subgroups of $G$, so a study of the finite geometries $R(\mathfrak{g}, A)$ reveals (quite different) subgroups of $G$.

I will discuss the classification of finite maximal tori in the classical case, and some interesting examples for the exceptional groups.