Affine Deligne-Lusztig varieties in either the affine Grassmannian or the affine flag variety are analogues of the Deligne-Lusztig varieties used to study finite groups of Lie type. The affine Deligne-Lusztig varieties arise naturally in a completely different context: the study of Shimura varieties over finite fields. It turns out that they encode some information on the fine structure of such Shimura varieties and related moduli spaces of $p$-divisible groups. In this talk I will explain some background and some recent work on the dimensions of affine Deligne-Lusztig varieties. This is joint work with U. Goertz, R. Kottwitz, and D. Reuman.