18.104, Seminar in Analysis, Fall 2001

Meeting time: Tuesday and Thursday 11–12:30, room 2-151

Text: Robert M. Young, *Excursions in Calculus: An Interplay of the Continuous and the Discrete*

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One way of describing analysis is as the study of inequalities: of finding upper and lower bounds for interesting things that you can't calculate exactly. In calculus classes, this theme can get lost because of the emphasis on things that you *can* calculate exactly. In 18.100, the inequalities are more in evidence, but it may not be so clear what it is you're trying to calculate.

Young's text emphasizes analytic examples coming from elementary number theory. Here is a typical one. Given a positive integer m, in how many ways can m be represented as a sum of squares of two integers? Write $r(m)$ for this number of ways. For example,

$$\begin{aligned} 5 &= 1^2 + 2^2 = (-1)^2 + 2^2 = 1^2 + (-2)^2 = (-1)^2 + (-2)^2 \\ &= 2^2 + 1^2 = (-2)^2 + 1^2 = 2^2 + (-1)^2 = (-2)^2 + (-1)^2, \end{aligned}$$

so $r(5) = 8$. On the other hand, some numbers cannot be written as sums of two squares in any way; $r(21) = 0$, for example. Writing an exact formula for $r(m)$ is difficult. A natural question for analysis is this: how big is $r(m)$ on the average? The answer is striking: the average of the first M values of r is approximately π, with an error smaller than $6M^{-1/2}$.

Some of the other topics to be covered in the seminar include Bernoulli numbers, and (therefore) explicit formulas for sums like

$$1^k + 2^k + 3^k + \cdots + n^k;$$

Stirling's approximation for factorials; and the arithmetic/geometric mean inequality.

The details of the structure of the seminar will depend on enrollment, but I hope that about two thirds of the material will be presented by the students. This might mean doing four or five half-hour presentations in the course of the semester. You'll have some control over when you speak, and over the material you present: typically a section of the text, or a solution of a family of problems from the text. For one of the presentations, you'll write five pages or more of lecture notes: something like a textbook account of the topic of your presentation. These notes will be distributed to the rest of the seminar participants in advance. After your presentation, you'll have a chance to incorporate improvements in your notes.

There will be some homework problems assigned. Grading will be based about one fourth on your written lecture notes, one fourth on homework, and half on participation: on your own presentations, and on your constructive criticisms of other presentations. There will be no exams or final.