April 1, 2011

18.01 Problem Set 9
Due Wednesday, April 13, in recitation

Collaboration and discussion of problem sets is a good idea; you must write up your answers on your own, and you must answer question 0 of Part II.

Part I: 10 points
Notation for homework problems: “2.4/13” means Problem 13 at the end of section 2.4 in Simmons. “1A-3” means Exercise 1A-3 in Section E (Exercises) of the Supplementary Notes.

1. 4J-1, 3, 6.
2. 5B-5, 9, 14; 5C-3, 5, 6, 14.

Part II: 15 points

0. Write the names of all the people you consulted or with whom you collaborated and the resources you used, beyond the course text and notes and your instructors; or say “none” or “no consultation.”

1a) Find the average value of the function \(1 = \cos^0(x)\) on the interval \(0 \leq x \leq 2\pi\).

b) Find the average value of the function \(\cos^2(x)\) on the interval \(0 \leq x \leq 2\pi\). (You can just quote an answer from the last problem set if you prefer.)

c) Find the average value of the function \(\cos^4(x)\) on the interval \(0 \leq x \leq 2\pi\).

d) Find the average value of the function \(\cos^6(x)\) on the interval \(0 \leq x \leq 2\pi\).

e) Explain why the answers for (a)–(d) are decreasing.

2) You can use the formulas

\[
\cos(ax) \cos(bx) = \frac{1}{2} \left( \cos((a + b)x) + \cos((a - b)x) \right)
\]

\[
\cos(ax) \sin(bx) = \frac{1}{2} \left( \sin((a + b)x) + \sin((a - b)x) \right)
\]

\[
\sin(ax) \sin(bx) = \frac{1}{2} \left( -\cos((a + b)x) + \cos((a - b)x) \right)
\]

to write things like \(\sin^M(x) \cos^N(x)\) as sums of terms like \(\cos(nx)\) and \(\sin(nx)\) (with \(n\) smaller than \(M + N\)).

a) Use this idea to find a formula of the form

\[
\cos^3(x) = A \cos(3x) + B \cos(2x) + C \cos(x) + D.
\]

b) Use the formula in (a) to calculate \(\int \cos^3(x) dx\).

c) Use this idea to say as much as you can about the trigonometric identity

\[
\cos^n(x) = a_n \cos(nx) + a_{n-1} \cos((n - 1)x) + a_{n-2} \cos((n - 2)x) + \cdots + a_1 \cos(x) + a_0.
\]

(Best answer is a formula for every coefficient \(a_n\). But if you can say something like, “the last term \(a_0\) is 11 when \(n\) is odd,” or “every fifth term is zero,” that’s good too.)