
1. Let $d, n$ be two integers. We say $d$ divides $n$, written $d|n$, if there is an integer $i$ such that $d \cdot i = n$. For example $7|1001$ since $7 \cdot 143 = 1001$ but $7 \nmid 101$ since 101 is prime. Since $d \cdot 0 = 0$ we always have $d|0$, no matter what $d$ is.

   (a) Prove $2|(n^2 + n)$ for all $n \geq 1$.
   (b) Prove $6|(n^3 + 5n)$ for all $n \geq 1$.
   (c) Prove that $2|(n^2 + n)$ and $6|(n^3 + 5n)$ for all integers $n$.

2. Prove the following identities:

   (a) $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n + 1)! - 1$ for all $n \geq 1$.
   (b) $\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n(n-1)} = 1 - \frac{1}{n}$ for all $n \geq 2$.

3. Show that as long as $x \geq -1$ then $(1 + x)^n \geq 1 + nx$ for all $n \geq 1$.

4. Prove that if you have an unlimited amount of 3 and 5 cent stamps, then you can make exact postage for $n$ cents for all $n \geq 8$. (Example: $n = 15 = 5 + 5 + 5$.)

5. Prove that $\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}$.