HW 2: assigned 24 Jan 2003, due 31 Jan 2003, hand in during class on stapled loose leaf paper

1. (a) Prove that \( \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n} \).
   
   (b) Prove that \( \binom{n}{0} + 2\binom{n}{1} + 4\binom{n}{2} + \cdots + 2^n\binom{n}{n} = 3^n \).

2. Suppose you record an ordered sequence of 10 votes, each vote either for candidate A or candidate B.
   
   (a) Suppose A wins and furthermore is winning at each stage in the sequence. How many ways can this happen?
   
   (b) Suppose B wins and furthermore is never losing at any stage in the sequence. How many ways can this happen?

3. Suppose you want to walk from the point \((0, 0, 0)\) to the point \((5, 4, 3)\) and you are only allowed to do this by repeatedly taking steps of the following three types: \((1, 0, 0)\), \((0, 1, 0)\), and \((0, 0, 1)\). Show that the total number of ways you can do this is the same as the coefficient of \(x^5y^4z^3\) in \((x + y + z)^{12}\).