18.02A Problem Set 2A – Fall 2009 due Thursday Nov.19, 12:45 in 2-106

Although this problem set is not due until after Exam 1, you should do it before the exam by way of exam preparation. The second part, Problem Set 2B, will be posted on the day of the exam, and will also be due on thursday, Nov.19.

Part I (15 points)

(You need not hand in the exercises in parentheses, which are just for more practice.)

Lecture 5. Thurs. Nov. 5 Parametric equations of lines and curves.
Read: 18.4, 17.1, 17.2 to middle p.593 Work: 1E-3bc,4; 1I-2b, 3ad, 5 (4,6)
Lecture 6. Friday. Nov. 6 Vector derivatives, **v**, **a**, **T**.
Read: 17.4 Work: 1J-1ac, 4abc, 6, 9.
Lecture 7. Tues. Nov. 10 Curvature; other applications.
Read: 17.5, Problem 1J-10 Work: 1J-3, 5, 10.
Lecture 8. Thurs. Nov. 12 Functions of several variables; partial derivatives, tangent plane.
Read: 19.1, 19.2, 19.3, Notes TA Work: 2A - 1abe, 2ae; 2B - 1b, 4, 6
Exam 1, Friday Nov.13 covering lectures 0-7, 2:05-2:55pm
Walker 3rd Floor (enter on river side)

Part II (25 points)

Problem 1. (Thurs. 3pts: 1+2)
Calculate the parametric equations for the line through $P_1 = (0, -1, 1)$ and $P_2 = (2, 1, 0)$. Use this parametric form to calculate its minimum distance from the origin.

Problem 2. (Thurs. 6pts: 2+2+2)
a) A jet takes off from (1, 1, 0) at time $t = 0$ and moves with constant speed $\mathbf{v} = (-5, 0, 1)$. In a flight simulator, the trajectory of the jet is displayed in the yz-plane as it would appear to an observer at the point (1, 0, 0). Find the formula (in the form $y = y(t), z = z(t)$) for the trajectory on the screen.
b) By considering the velocity vector of the point on the screen, show that the motion in part (a) is in a straight line. What happens as $t \to \infty$?
c) Draw a picture of several trajectories as they would appear on the screen if all of them are parallel to the same vector in 3-space.

Problem 3. (Friday 6pts: 2+2+2)
A hockey puck of radius 1 slides along the ice at a speed $10\sqrt{2}$ in the direction of the vector (1,1). As it slides, it spins in a counterclockwise direction at 2 revolutions per unit time. At time $t = 0$, the puck's center is at the origin (0,0).
a) Find the parametric equations for the trajectory of the point P on the edge of the puck initially at (1,0).
b) Find the velocity **v** of the point P.
c) What is the minimum speed of the point P, and what is the direction of the velocity at the corresponding time.

Problem 4. (Friday 3pts)
Find unit tangent and normal vectors to the parabola $(x, y) = (at^2, 2at)$ where a is a constant.

Problem 5. (Tues. 4pts: 1+1+1+1)
Consider the helical trajectory with displacement vector $\mathbf{r}(t) = \sin 4t\ \mathbf{i} + \cos 4t\ \mathbf{j} + 3t\ \mathbf{k}$, where t is time. Calculate
a) the velocity vector **V** and the unit tangent vector **T**,
c) the speed ds/dt and the arclength traced out between the points at $t = 0$ and $t = 2\pi$,
d) the curvature κ.
e) Show that the curve makes a constant angle with the vertical (**k**) direction.

Problem 6. (Thurs. 3 pts) Show that $z = \tan^{-1}(y/x)$ satisfies $z_{xx} + z_{yy} = 0$.