18.02A Problem Set 1 – Fall 2009 due Thursday 11/5/09, 12:45 in 2-106

18.02A Supplementary Notes: purchase in CopyTech, basement of Building 11.

Part I (20 points)

Do not hand in the exercises in parentheses, which are for more practice if you want or need it; hand in all the others.

Below, the notation **1A-2** means: Exercise 2 in Section 1A of the Exercises portion of the 18.02 Notes; it is solved in the Solutions section of the Notes. 17.3=section 17.3 of the textbook (Simmons 2nd ed.)

Recitation. Mon. Oct. 26. Intro. to vectors; addition, scalar multiplication.
Read: 17.3, 18.1   Work: 1A-4a,7bc,8ab,11 (1,2,6)

Lecture 20. Tues. Oct. 27.   Vectors; dot (or scalar) product.
Read: 18.2   Work: 1B-1a, 3a, 5b,11,13

Lecture 21. Thurs. Oct. 29   Low-order determinants; cross (or vector) product.
Read: 18.02 Notes D, pp. 1-3; 18.3   Work: 1C-2b,3b,4,9; 1D-1b,2,5,6

Lecture 22. Fri. Oct. 30.   Matrices; inverse matrices.
Read: Notes M.1, M.2   Work: 1F-5b, 8a; 1G-3, 4, 5

Lecture 23. Tues. Nov.3   Theorems about square systems; Equations of planes
Read: Notes M.3, M.4   Work: 1H-3abc, 7 . Read: pp. 648,9 Work: 1E-1cd, 2

Lecture 24. Thurs. Nov.5 Parametric equations for lines and curves; cycloid
Read 18.4, 17.1

Part II (30 points)

Directions. Try each problem alone for 15 minutes. If you subsequently collaborate, solutions must be written up independently. It is illegal to consult problem sets from previous semesters. With each problem is the day it can be done.

Problem 1. (Tues. 3 pts) A frog with a steady speed U mph in still water must travel straight across a river flowing with uniform speed of V mph. In what direction should the frog swim, and what is its actual speed and velocity? What speed is required to make such a crossing?

Problem 2. (Tues. 4 pts) Prove that the lines joining the midpoints of the opposite edges of a tetrahedron intersect and bisect each other.

Problem 3. (Thurs. 5 pts) A vertical fence stands on the ground, and the sun is shining on it. The ground is the xy-plane, and the top of the fence is the line through the point (0,0,1) in the direction of $< 3, 1, 0 >$. The sun's rays are pointing in the direction of the vector $< 2, -1, -3 >$. Find the region on the ground that is shaded by the fence.

Problem 4. (Thurs. 4 pts: 2+2) The Biot-Savart law, which you will study in 8.02, says that the magnetic field $\mathbf{B}$ at a point with position vector $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} = (x, y, z)$ induced by a current of magnitude J passing through the origin $(0, 0, 0)$ in the $\hat{\mathbf{k}}$-direction is given by

$$\mathbf{B} \;=\; \left(\frac{\mu_0}{4\pi}\right) J \,\frac{\hat{\mathbf{k}} \times \mathbf{r}}{r^3} \quad ,$$

where μ_0 is a physical constant and $r = \;|\,\mathbf{r}\,|$.
a) Find the formula for the magnetic field $\mathbf{B}$ at a general point (x, y, z) and evaluate it at the point $(1, 2, 3)$.
b) For which points (x, y, z) at unit distance from the origin is the magnitude of the magnetic field, $|\,\mathbf{B}\,|$, largest?

Problem 5. (Fri. 5 pts: 1+1+1+2) Consider the three vectors $\mathbf{A} =< 1, 0, 3 >$, $\mathbf{B} =< 1, 1, 0 >$ and $\mathbf{C} =< 0, 2, 1 >$.
a) What is the angle between vectors $\mathbf{A}$ and $\mathbf{B}$?
b) What is the area of the parallelogram with sides $\mathbf{A}$ and $\mathbf{B}$?
c) What is the volume of the parallelpiped with concurrent edges $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$?
d) By evaluating in turn the left- and right-hand sides, show that Lagrange's formula is satisfied:
$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Problem 6. (Fri. 5 pts: 3+2)
Four vectors are erected perpendicular to the four faces of a general tetrahedron. Each vector is pointing outwards and has a length equal to the area of the face. Show that the sum of these four vectors is zero.
Hint: let $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ be vectors representing the three edges starting from a fixed vertex. Express each of the four vectors in terms of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, and show that their sum is the zero vector. Do not introduce a coordinate system.
b) Formulate and prove the analogous statement for a plane triangle.

Problem 7. (Tues. 4 pts) For what value of λ do the four points $(0, -1, -1)$, $(3, 9, 4)$, $(-4, 4, 4)$ and $(4, 5, \lambda)$ lie in a plane?