Problem 1.
a) Write down in xy-coordinates the vector field $\mathbf{F}$ whose vector at $(x, y)$ is obtained by rotating 90° counterclockwise the radially-outward-pointing unit vector at $(x, y)$.

b) Let $\mathbf{F}$ be the field in part (a). Let $C_1$ be the line segment running from (1,1) to (2,2), and $C_2$ the positively-oriented circle of radius $a$ centered on the origin. Using intuition, give the value of the following (short answer; no calculation required):

i) $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$,  

ii) $\oint_{C_2} \mathbf{F} \cdot d\mathbf{r}$,  

iii) flux of $\mathbf{F}$ across $C_1$,  

iv) flux of $\mathbf{F}$ across $C_2$.

Problem 2.
Let $\mathbf{F} = \nabla f = \text{grad} f$, where $f(x, y) = x^2 + 4y^2$.

a) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $C$ is a curve running from (1,1) to (2,2).

b) Find the locus of all points $(x, y)$ in the plane such that $\int_{(1,1)}^{(x,y)} \mathbf{F} \cdot d\mathbf{r} = 0$.

Problem 3.
Let $\mathbf{F} = y(ax + y)i + (3x^2 + bxy + y^3)j$, where $a$, $b$ are constants.

a) Prove: if $\mathbf{F}$ is conservative, then $a = 6$, $b = 2$. (Use these values in part b).

b) Using a systematic method (show work), find a function $f(x, y)$ such that $\mathbf{F} = \nabla f$.

Problem 4.
Let $C$ be the portion of the parabola $y = 1 - x^2$ lying over the $x$-axis, oriented in the direction of decreasing $x$. Taking

$$\mathbf{F} = (6xy^5)i + (1 + x^2y - y^6)j,$$

a) set up an integral in $x$ alone that represents the flux of $\mathbf{F}$ over $C$. (Give integrand and limits, but do not evaluate);

b) calculate the flux of $\mathbf{F}$ over $C$ by using Green’s Theorem in the normal form. (Note that $C$ is not closed).

Problem 5.
Show that the value of $\oint_C (y^2 - 2y) \, dx + 2xy \, dy$ around a positively oriented circle $C$ depends only on the size of the circle, and not on its position.

Problem 6.
Consider the integral $\int\int_R (x + y)^4(3x - y)^4 \, dxdy$, where $R$ is the triangle with vertices at $x = -1$ and $x = 3$ on the $x$-axis, and $y = 3$ on the $y$-axis.

Let $u = x + y$ and $v = 3x - y$. Express the double integral in $uv$-coordinates; use as the order of integration $dv \, du$. 