**Problem 1.**

a) In the $xy$-plane, let $F = Pi + Qj$. Give in terms of $P$ and $Q$ the line integral representing the flux of $F$ across a simple closed curve $C$, with outward-pointing normal.

b) Let $F = axi + byj$. How should the constants $a$ and $b$ be related if the flux of $F$ over any simple closed curve $C$ is equal to the area inside $C$?

**Problem 2.**

A solid hemisphere of radius 1 has its lower flat base on the $xy$-plane and center at the origin. Its density function is $\delta = z$. Find the force of gravitational attraction it exerts on a unit point mass at the origin.

**Problem 3.**

Evaluate $\int_C (y - x)dz + (y - z)dx$ over the line segment $C$ from $P : (1, 1, 1)$ to $Q : (2, 4, 8)$.

**Problem 4.**

Consider a solid sphere of radius $a$ with center at the origin; let $H$ be its solid upper hemisphere (i.e., the part above the $xy$-plane). Set up a triple integral in spherical coordinates which gives the average distance of a point in $H$ from the $xy$-plane. (Give integrand, limits, and the constant factor in front, but do not evaluate.)

**Problem 5.**

Let $C$ be a solid right circular cone having base radius 1 and vertex angle $60^\circ$. Set up an integral in cylindrical coordinates which represents the moment of inertia of $C$ about its central axis; assume the density $\delta = 1$. (Place the cone so its axis is the $x$-axis and its vertex is at the origin; supply integrand and limits, but do not evaluate.)

**Problem 6.**

a) Let $F = ay^2i + 2y(x + z)j + (by^2 + z^2)k$. For what values of the constants $a$ and $b$ will $F$ be conservative? Show work.

b) Using these values, find a function $f(x, y, z)$ such that $F = \nabla f$.

c) Using these values, give the equation of a surface $S$ having the property: $\int_C F \cdot dr = 0$ for any two points $P$ and $Q$ on the surface $S$.

**Problem 7.**

Let $S$ be the surface formed by the part of the graph of the paraboloid $z = x^2 + y^2$ lying below the plane $z = 1$, and let $F = xi + yj + (1 - 2z)k$.

Calculate the flux of $F$ across $S$, taking the outward direction (i.e., the one pointing away from the $z$-axis) as the one for which the flux is positive. Do this two ways:

a) by a method which calculates $\int_S F \cdot dS$ directly;

b) by using the divergence theorem.

**Problem 8.**

Let $S$ be the infinite circular cylindrical surface given by the equation $x^2 + y^2 = 1$ having the whole $z$-axis as its central axis, and let $F = (xz - y)i + xyj + zk$.

a) Calculate $\nabla \times F$ (i.e., curl $F$).

b) Deduce that $\iint_R \nabla \times F \cdot \mathbf{n} \, dS = 0$ for any finite portion $R$ of the surface $S$.

c) Let $C$ be any closed curve on $S$ going once around $S$ (and oriented as in the picture). Show by using the result of part (b) and Stokes' theorem that $\oint_C F \cdot dr$ always has a constant value independent of $C$, and determine this value.

**Problem 9.**

Let $\phi(x, y, z)$ be a function with continuous second partial derivatives. Prove that $\nabla \times \nabla \phi = 0$.

**Problem 10.**

An $xz$-cylinder in 3-space is a surface given by an equation $f(x, z) = 0$ in $x$ and $z$ alone; its section by any plane $y = c$ perpendicular to the $y$-axis is always the same $xz$-cylinder. (See picture.)

Show that if $F = x^2i + y^2j + xzk$, then $\iint_S F \cdot \mathbf{n} \, dS = 0$ for any simple closed curve $C$ lying on an $xz$-cylinder. (Use Stokes' theorem.)