

18.901
Homework #10 Extra Problem

This problem is designed to make you think about the proof of the Urysohn lemma. The notation will be as in the proof of the Urysohn lemma in the book (which should be very similar to the notation I used in my lecture). Let

$$\begin{aligned}X &= [0, 2], \\A &= \{0\}, \\B &= \{2\}.\end{aligned}$$

We wish to use the lemma to construct a function $f : X \rightarrow [0, 1]$ such that $f|_A = 0$ and $f|_B = 1$. For $q \in \mathbb{Q}$, define U_q as follows. For $q < 0$, set $U_q = \emptyset$. For $q = 1$, set $U_q = [0, 1)$. For $q \in \mathbb{Q} \cap (0, 1)$, define $U_q = [0, 1 + q)$.

Question : What function will result when you run the proof of Urysohn's lemma?

Remark : This should also convince you that the function you get will achieve non-rational values.