Harvard-M.I.T. Algebraic Geometry Seminar

MORPHISMS OF HYPERSURFACES

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Abstract:

Given a nonconstant morphism $f : X_d \to Y_e$ of smooth hypersurfaces of the indicated degrees in $\mathbb{P}^n$, with $n \geq 4$, the Grothendieck-Lefshcetz Theorem says that $f^*\mathcal{O}_Y(1) = \mathcal{O}_X(m)$ for some positive integer m. What can we say about m in terms of d and e?

One possibility is that e divides d and $m = d/e$. This simply means that f extends to a morphism on all of $\mathbb{P}^n$, and X is the preimage of Y under this extended morphism. To my knowledge, this is the only possibility. We will enumerate some cases where we can show that $m = d/e$ is indeed the only possibility for m.

First we will show that m is bounded in terms of d and e. Then we will extend f to a rational map on $\mathbb{P}^n$ and consider the preimage of Y, which is X plus some other hypersurface H. In case $m \neq d/e$, i.e. H is not empty, we will analyze the rational map from H to Y and derive a contradiction in some cases. In particular, we will show that if $f : X_d \to Y_3$ is a morphism of hypersurfaces in $\mathbb{P}^4$ and $d \leq 5$, then f is either constant or $d = 3$ and f is an isomorphism.

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