Harvard-M.I.T. Algebraic Geometry Seminar

K-THEORY OF A HENSELIAN DISCRETE VALUATION FIELD WITH NON-PERFECT RESIDUE FIELD

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Abstract:

Twenty-five years ago, Bloch introduced the complex of p-typical curves on Quillen's algebraic K-groups for the purpose of calculating crystalline cohomology. This lead Deligne and Illusie to define the de Rham-Witt complex, which gives crystalline cohomology. I have showed that for a regular scheme over $\mathbb{F}_p$, the two complexes, in fact, are isomorphic. Both complexes can be defined also for schemes over a discrete valuation ring V of mixed characteristic. And in this case, they are not the same. In this talk, I will explain the structure of the two complexes for a smooth V-scheme X. Let me just mention here that the Frobenius fixed set of the de Rham-Witt complex (modulo p) is isomorphic to the sheaf of p-adic vanishing cycles.

The calculation of Bloch's p-typical curves in the mixed characteristic setting has the following consequence for algebraic K-theory (whence the title): Let K be the quotient field of the henselian local ring of X at the generic point of the special fiber. Then, assuming that $\mu_p \subset K$, there is a canonical isomorphism

$$K_*^M(K) \otimes_{\mathbb{Z}} S_{\mathbb{Z}/p}(\mu_p) \xrightarrow{\sim} K_*(K, \mathbb{Z}/p),$$

which to $\zeta \in \mu_p$ assigns the corresponding Bott element $b_\zeta \in K_2(K, \mathbb{Z}/p)$. This is the value of the K-groups predicted by the Beilinson-Licthenbaum conjectures.

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