Harvard-M.I.T. Algebraic Geometry Seminar

MODULI SPACES OF ABELIAN VARIETIES IN POSITIVE CHARACTERISTIC

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Abstract:

Abelian varieties in characteristic p have additional structures (the action of Frobenius on the p-divisible group, Newton polygons, structures of the p-kernel) which give access to geometric aspects of these moduli spaces in all characteristics.

Here is one motivating

CONJECTURE. For g>2 the moduli space of abelian varieties of dimension g over the complex numbers does not contain a compact codimension g subvariety.

In this talk we discuss various structures, constructions, theorems, conjectures, and we indicate interrelations between the following aspects:

(GC) A conjecture by Grothendieck on the realization of Newton Polygons in families of p-divisible groups (Grothendieck, Montreal 1970, now a theorem). One essential ingredient for this proof is the "Purity theorem" (joint work with Johan de Jong). As a corollary: a new proof of the conjecture by Manin that every symmetric NP is realized by an abelian variety.

(NP) The stratification by Newton Polygons of the moduli space of abelian varieties, and of deformation spaces of p-divisible groups. We compute the dimension of these strata.

(EO) The stratification by the structure of p-kernels (joint work with Torsten Ekedahl). This stratification is now widely studied in various disguises. As a corollary: a new proof of the irreducibility of the moduli space of principally polarized abelian varieties in arbitrary characteristic.

(Fol) Two different "foliations" of NP-strata. Conjecturally these describe the Zariski-closure of Hecke orbits in positive characteristic. These structures give access to a precise description of an "almost product-structure" of these leaves covering an open NP stratum. This structure is used by Elena Mantovan in a computation of the cohomology of certain Shimura varieties of PEL type.

(p-Div) Possibly we will discuss new results by Thomas Zink on families of p-divisible groups.

March 5, 2002 3:00 p.m. Harvard Room 507