

# Harvard-M.I.T. Algebraic Geometry Seminar

## Perverse sheaves and dualizing complexes over noncommutative ringed schemes

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### Abstract:

In this talk I will discuss an attempt at Grothendieck Duality on noncommutative spaces. Since in the case of affine noncommutative spaces (i.e. rings) Grothendieck Duality is pretty well understood, and on the other hand we don't even know what is a noncommutative space in general, we consider an intermediate case: a noncommutative space  $Y$  that's an affine fibration over a commutative scheme  $X$ . That's a fancy way to say that  $Y = (X, A)$  where  $A$  is a sheaf of quasi coherent noncommutative rings on  $X$ . We call  $(X, A)$  a quasi coherent ringed scheme.

As usual in such circumstances, we encounter the problem of gluing. On each affine open set  $U$  in  $X$  we have a rigid dualizing complex for  $A|_U$  from the ring construction, and these are compatible on intersections. But how to glue these complexes globally? One should note that Cousin complexes, the solution devised by Grothendieck for gluing dualizing complexes, will not work in the noncommutative world due to well known obstructions.

Instead we choose to use perverse sheaves. This is a gluing method invented by Bernstein-Beilinson-Deligne-Gabber in the context of geometry of singular spaces. We discovered that the Auslander condition of dualizing complexes over noncommutative rings (a very algebraic property) is exactly what is needed to define perverse modules over a noncommutative ring. And furthermore using a few nice features of the theory, we can also extend the definition from rings to noncommutative ringed schemes. Finally it turns out that rigid dualizing complexes are themselves perverse sheaves of bimodules (namely on the product  $X^2$ ), so we can glue the local pieces together.

I will explain what are dualizing complexes and what they are good for (concentrating on the noncommutative side). Then I'll discuss perverse sheaves, the Auslander condition and how they interact. I'll finish by sketching our construction.

The work is joint with J.J. Zhang (Seattle).

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3:00 p.m.

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