Evaluation and Comparison of Continuum Models for Dense Granular Flow

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Dense Flow Models:

1. Mohr-Coulomb Plasticity
   [Sokolovskii (1965), Jenike & Johanson (1962)]

2. Hourglass Theory
   [Savage (1967), Sullivan (1972), Davidson & Nedderman (1973)]
3. Kinematic Model
   [Litwiniszyn (1963), Nedderman & Tuzun (1979)]

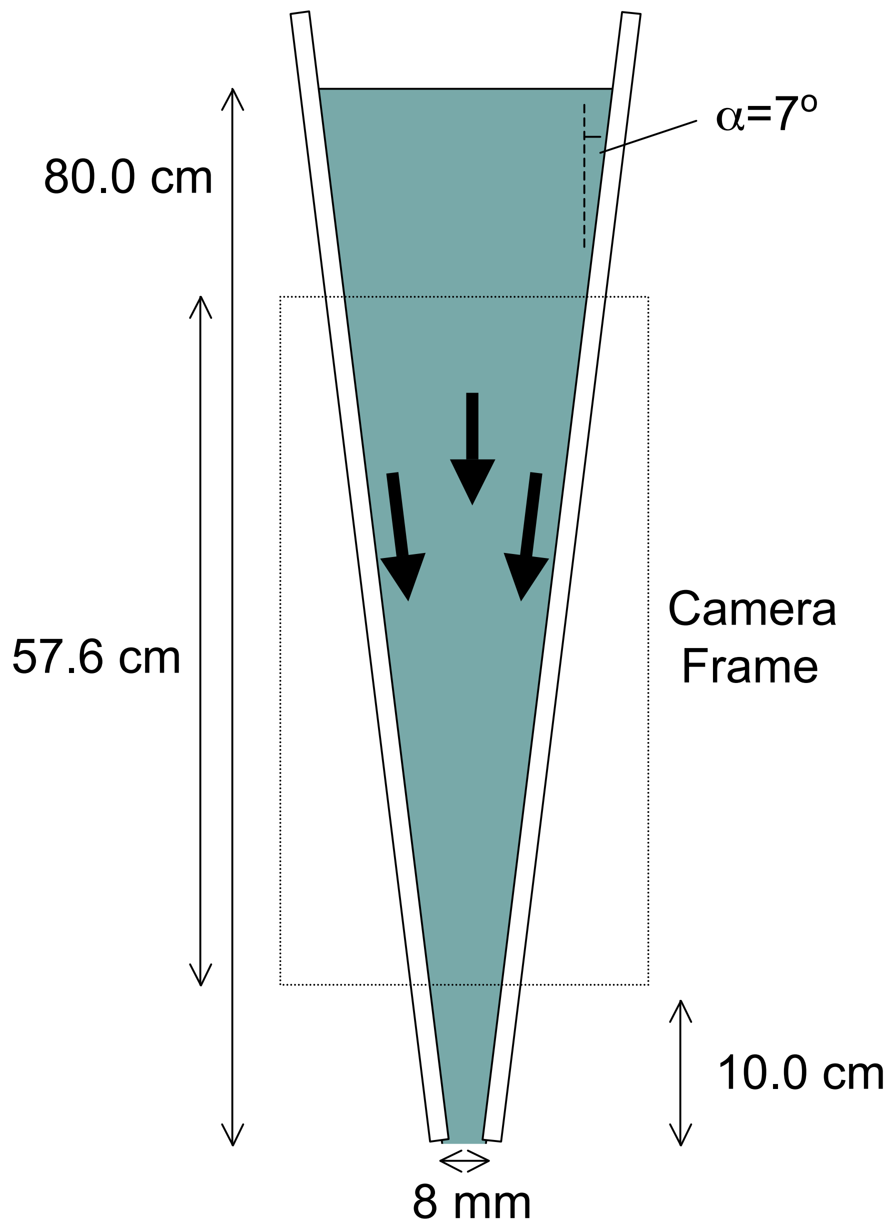
Setup

Grains: 3 mm glass spheres

Side Walls: smooth and fully rough

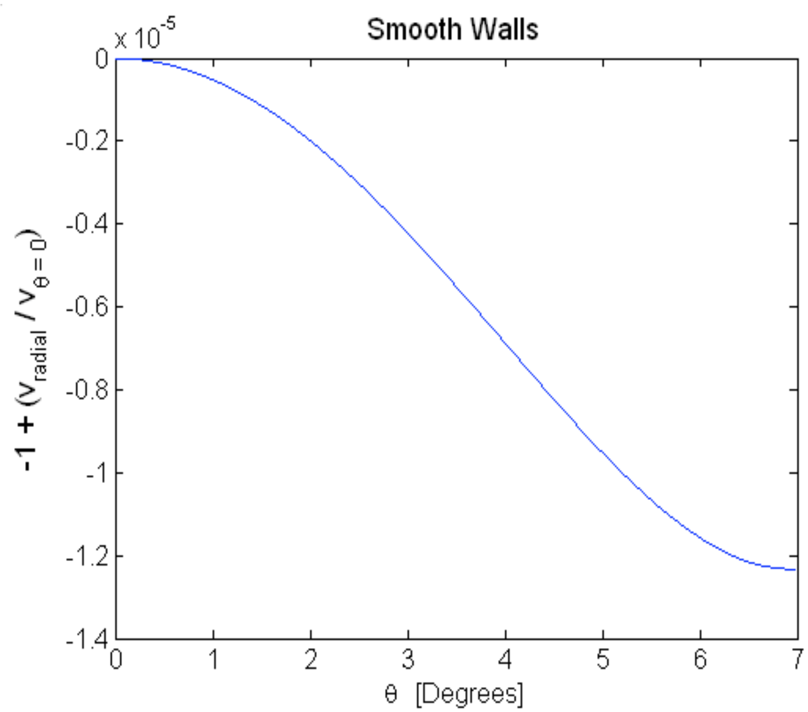
Depth: 2.5 cm

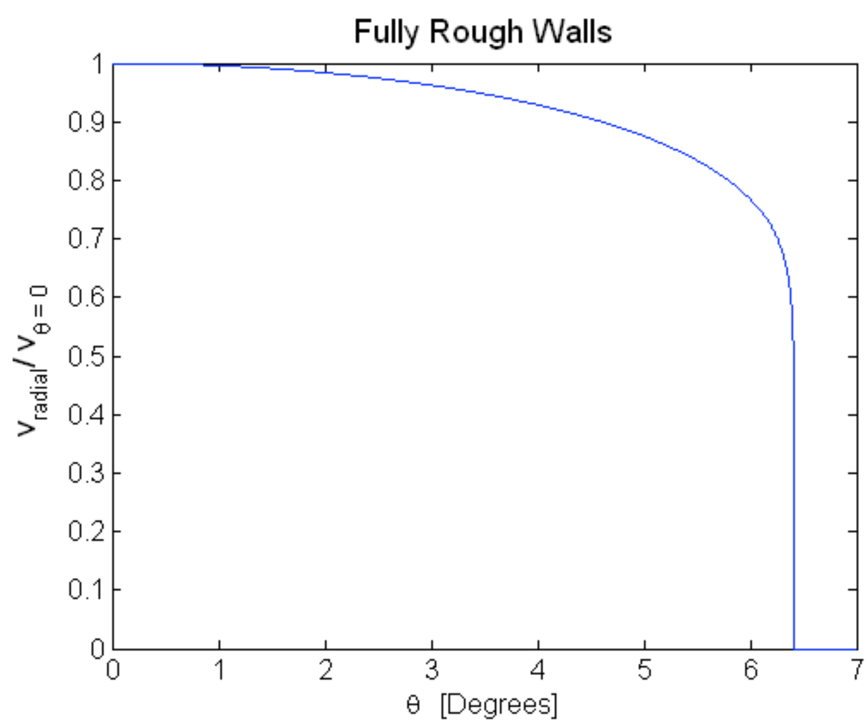
Camera: 250 fps



Mohr-Coulomb Plasticity

- Yield occurs only when $\tau = \mu_{internal}\, \sigma = (\tan \phi)\, \sigma$. For us, $\phi = 24.5^{o}$.
- Material assumed to be at incipient yield everywhere and quasi-static.
- Upholds Levy flow rule.
- Jenike found a similarity solution operative in a wedge geometry which gives radial flow varying in θ:





Hourglass Theory

- Least complicated flow model to apply.
- Based on M-C Plasticity with simplifications to enable convective solution.
- Only for wedge or conical hopper geometry with small apex angle.
- Requires hopper walls to be frictionless. Asserts that flow is radial and constant in θ.

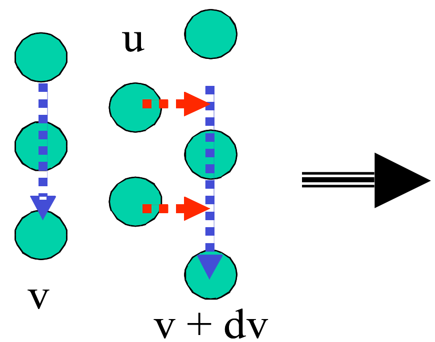
Result for narrow wedge: $\boxed{\vec{v} = -\frac{A}{r}\hat{r}}$

for $A = \sqrt{\frac{(K+1)(r_0^{K+1}r_1^3 - r_0^3 r_1^{K+1})g}{(K-2)(r_0^{K+1} - r_1^{K+1})}}$, $K = \frac{1+\sin\phi}{1-\sin\phi}$.

Thus we have, $\boxed{A = 370.0\,\mathrm{cm}^2\mathrm{s}^{-1}}$.

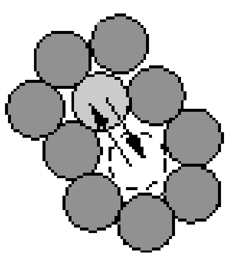
Kinematic Model

General principle:

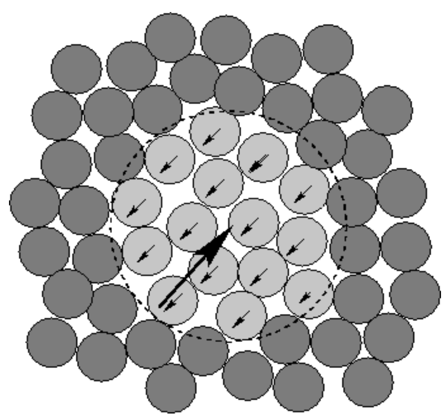


$$u = b\,\frac{\partial v}{\partial x}\ ,\qquad \frac{\partial v}{\partial z} = b\,\frac{\partial^2 v}{\partial x^2}$$

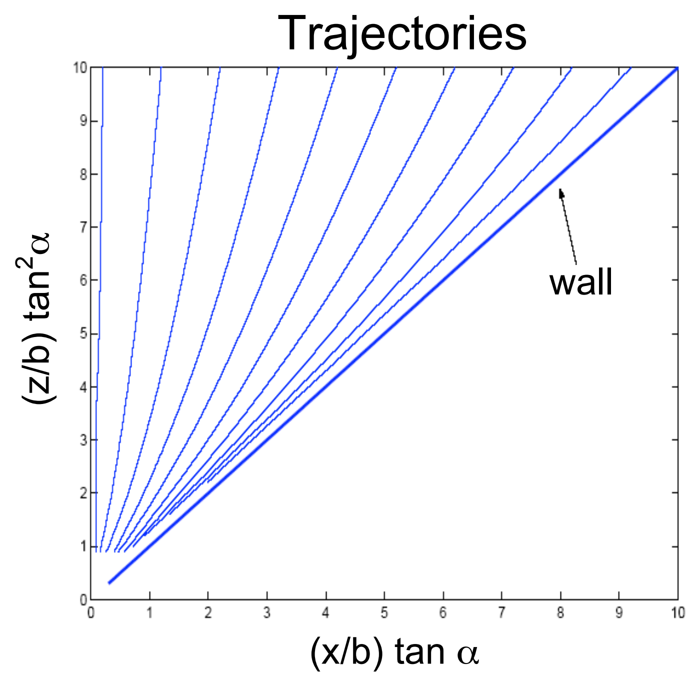
- Only for use in silos and hoppers.
- No dependence on internal or wall friction.
- Flow governed by boundary conditions and empirically determined diffusion length b.



Void model



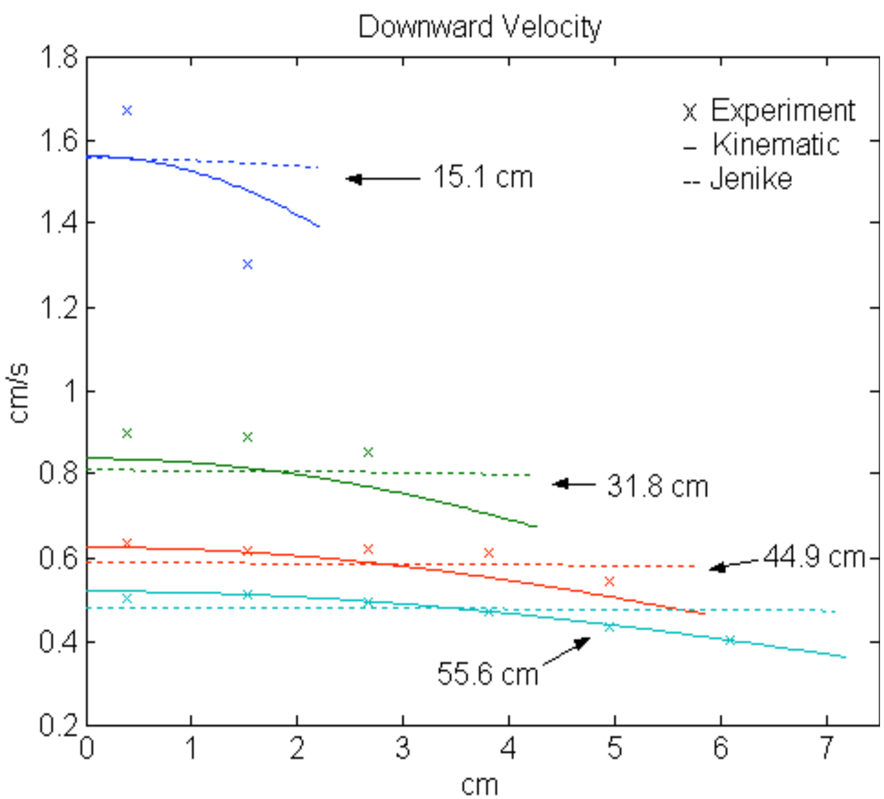
Spot model

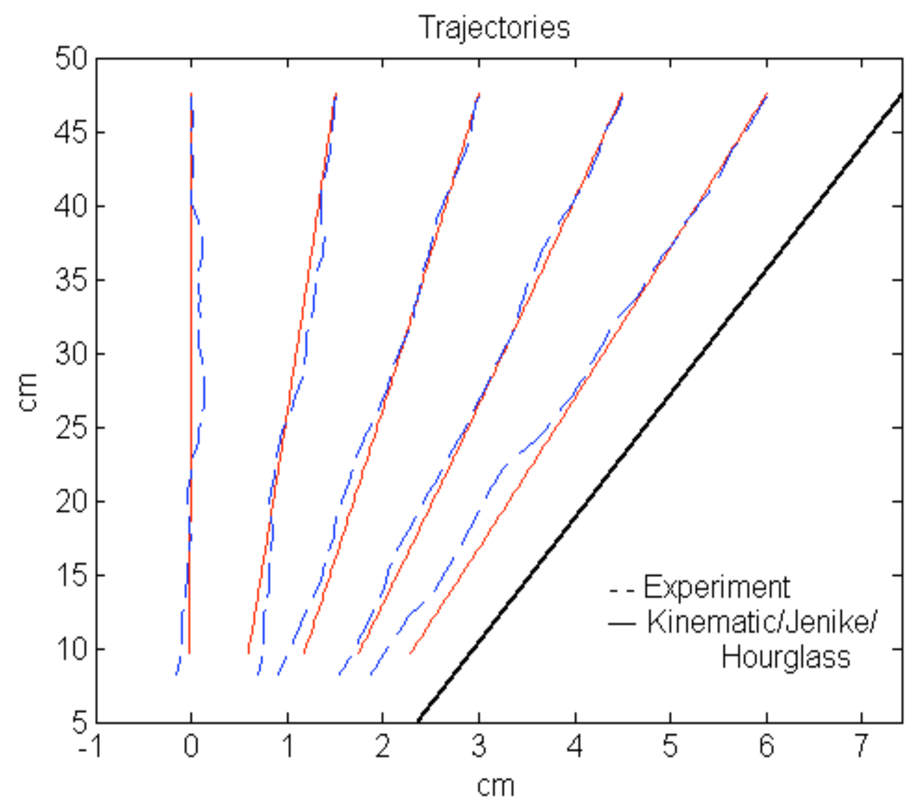


Experiment: Smooth Walls

Hourglass Constant:
A = 370.0 cm^2/s theoretically
A = 25.0 cm^2/s experimentally

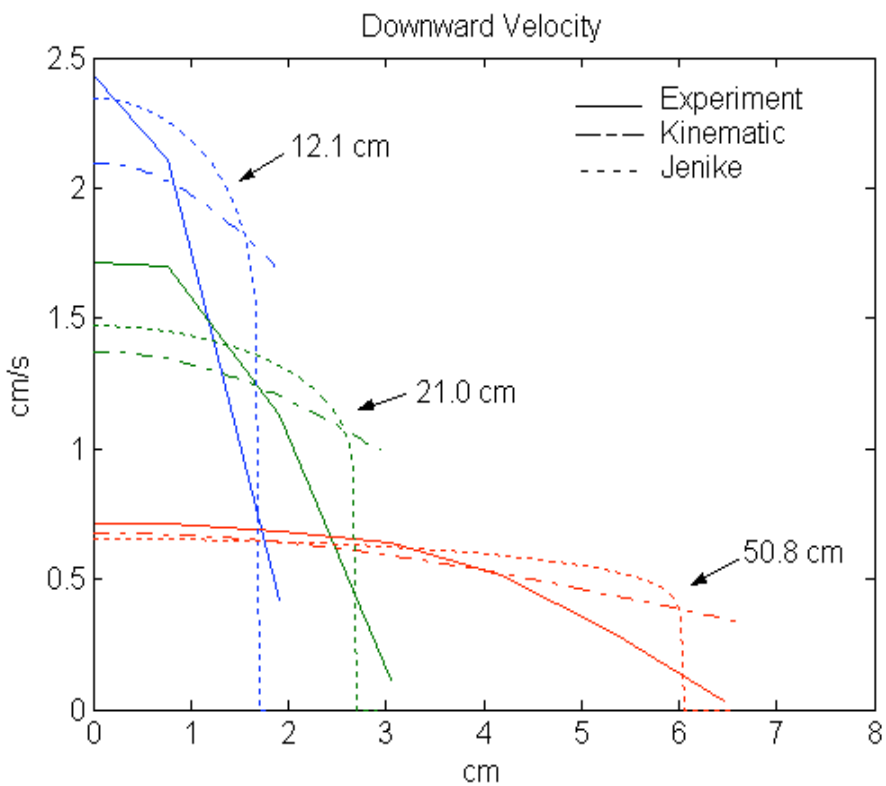
Kinematic Diffusion Length:
b = 3.86d

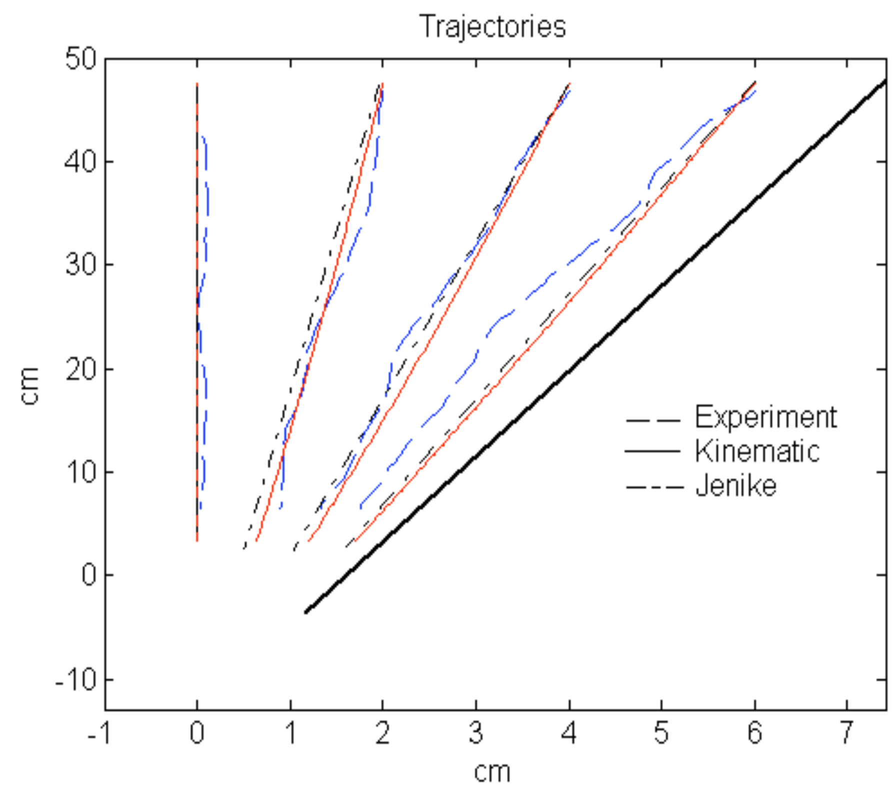




Experiment: Fully Rough Walls

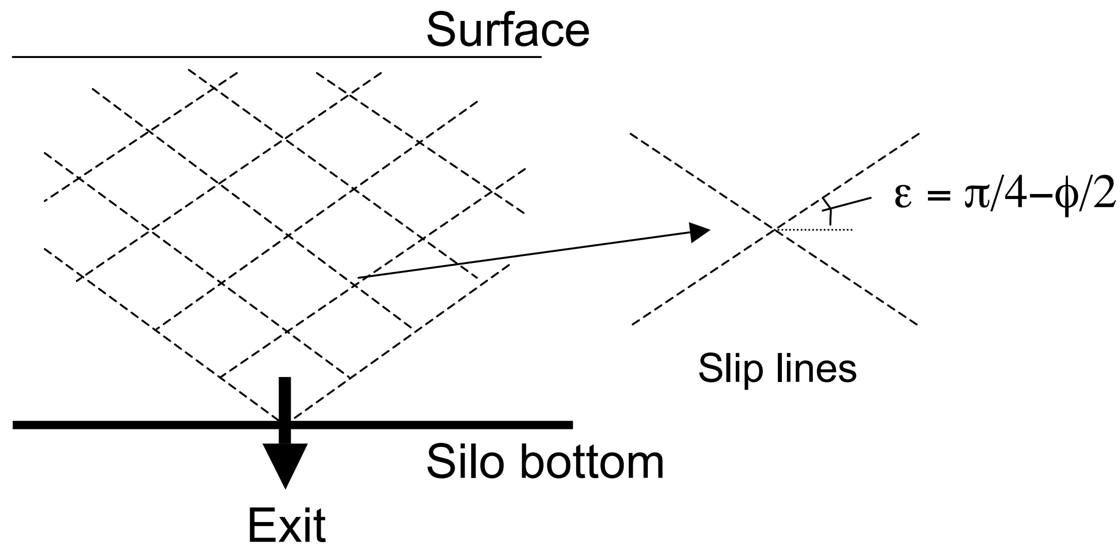
Kinematic Diffusion Length: b = 1.77d



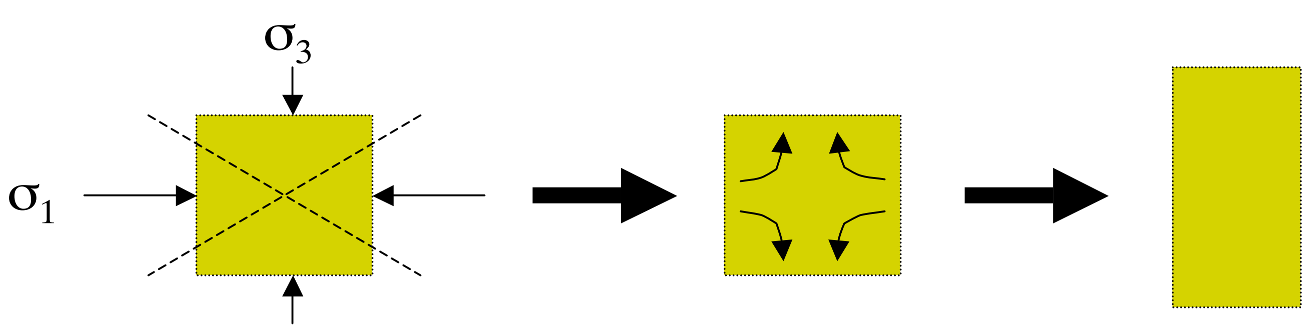


Possible Connections

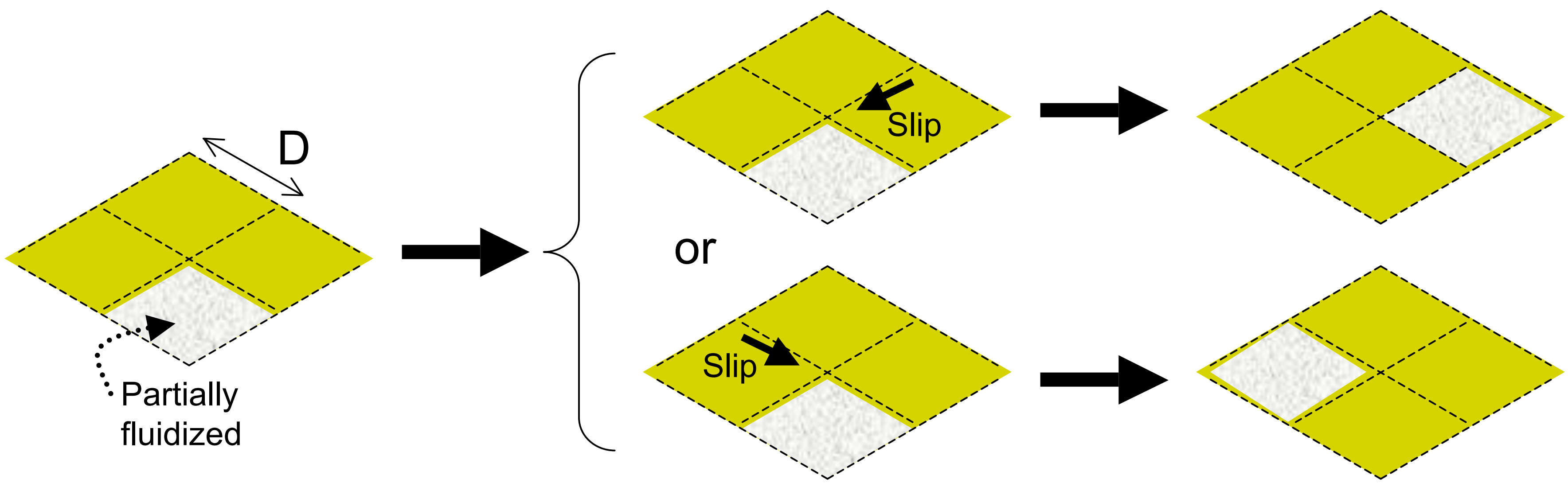
In a wide, approximately 2D silo, solution to Mohr-Coulomb slip lines turn out to be straight lines along $|\theta| = \varepsilon$ extending down from the surface.

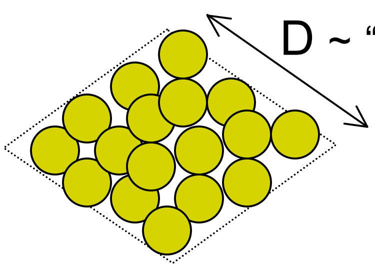


Co-axiality (Levy Flow Rule):



Stochastic Flow Rule





Typical cell

Rule produces analytic formula for Kinematic b:

$$b = D\cos^2\epsilon/(2\sin\epsilon)$$

For glass beads: $20^o < \phi < 25^o$, $3d < D < 5d$, predict $1.75\ d < b < 3.31\ d$. (Experiments: b= 1.3d - 3.4d.)

Conclusions

1) While each model accounts well for certain effects, none alone can accurately predict a dense flow with generality.

2) The Kinematic Model, successful thus far as an empirical model, may be mechanically derivable from the Mohr-Coulomb stress equations but with a stochastic flow rule. May enable us to generalize the use of Kinematic Modeling beyond drainage.