The Ground Axiom

Jonas Reitz
(CUNY – Graduate Center)

Since the invention of forcing by Paul Cohen in 1962 set theorists have demonstrated the flexibility of the technique by using it to produce an enormous variety of models of set theory. The models thus obtained, though diverse, do have a common feature: they are all forcing extensions. This suggests a natural question - under what circumstances is the universe not a forcing extension? A new axiom is proposed, the Ground Axiom (GA), asserting that the universe is not a forcing extension of any inner model. While this axiom is second order in nature, I will show that it is expressible as a first order statement. That GA is consistent follows from observing that the axiom holds in L, Goedel's constructible universe. I will discuss a flexible method for constructing models of GA, and show that any model of ZFC has an extension which satisfies GA. In addition, for a fixed model V that does not satisfy the Ground Axiom one can consider those inner models from which V may be obtained by forcing. This collection, which I call the ground models of V, is amenable to first-order analysis and a wealth of questions arise concerning its possible properties. I will discuss some results and open questions in this area.